Система (6) дополняет список систем Спротта [2], обладающих (при определенных значениях параметров) хаотическим поведением.

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DIFFERENTIAL EQUATIONS VS POWER SERIES

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A typical approach for solving ordinary differential equations with variable coefficients is to seek their solutions in the form of a generalized power series. However, we may wish to know more about the properties of coefficients in the series, such as their partial sums or weighted sums. We work with a general second-order linear differential equation

$$y'' + a(z)\,y' + b(z)\,y = 0, \tag{1}$$

where $a(z)$ and $b(z)$ are continuous functions on some interval. Suppose $y(z)$ is a series solution of this equation. If $y(z)$ has a Maclaurin representation $y(z) = \sum_{n\geqslant 0} c_n z^n$, then the series is a generating function for its sequence of coefficients $\{c_n\}_{n\geqslant 0}$. The sequence of finite sums $\sigma_n = \sum_{k=0}^{n} c_k$ has generating function [1] given by:

$$S(z) = \frac{y(z)}{1-z} = \sum_{n\geqslant 0}\left(\sum_{k=0}^{n} c_k\right) z^n = \sum_{n\geqslant 0} \sigma_n\, z^n.$$

Actually, the function $S(z)$ satisfies a differential equation

$$S''(z) + \left(a(z) - \frac{2}{1-z}\right) S'(z) + \left(b(z) - \frac{a(z)}{1-z}\right) S(z) = 0.$$

As illustration, consider Chebyshev's equation (in the variable x) $(1-x^2)y''-xy'+n^2y=0$, where n is a positive integer. This equation has the form of (1) with $a(x) = -x/(1-x^2)$ and $b(x) = n^2/(1-x^2)$. It has two linearly independent solutions $T_n(x)$, known as the Chebyshev polynomial of the first kind, and $\sqrt{1-x^2}U_{n-1}(x)$, where $U_{n-1}(x)$ is the Chebyshev polynomial of the second kind (of degree $n-1$). The polynomial $T_n(x)$ can be considered as a generating function for its coefficients, which are zero starting with index $n+1$. Let $\sigma_{k,n}$ be the sum of all coefficients up to index k of $T_n(x)$ $(n = 0, 1, 2, \ldots, k = 0, 1, \ldots n)$. Obviously, this sequence stabilizes when k exceeds n : $\sigma_{n,n} = \sigma_{n+1,n} = \sigma_{n+2,n} = \ldots$. Moreover, the sum of all coefficients in any Chebyshev polynomial $T_n(x)$ is 1, which follows from the relation $(1-x)^{-1}T_n(x) = P_{n-1}(x) + (1-x)^{-1}$, where $P_{n-1}(x)$ is a polynomial of degree $n-1$. Similarly, from the relation $U_{n-1}(x) = Q_{n-2}(x)(1-x)+n$, for some polynomial $Q_{n-2}(x)$ of degree $n-2$, it follows that the sum of all coefficients in Chebyshev polynomial of the second kind $U_{n-1}(x)$ is n.

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