

a  $p$ -adic solenoid and denoted by  $\Sigma_p$  (see [1]). As a set with a measure  $\Sigma_p \cong [0, 1) \times \mathbb{Z}_p$ . It is a compact group and has a natural measure  $dx \cdot d_p u$ . Pontryagin's dual group of the  $p$ -adic solenoid is  $\widehat{\Sigma}_p = \mathbb{Q}^{(p)} = \bigcup_{n=0}^{\infty} p^{-n}\mathbb{Z}$ . That means any  $f \in L_2(\Sigma_p)$  can be expanded into a Fourier series

$$f(x, u) = \sum_{\alpha \in \mathbb{Q}^{(p)}} \widehat{f}(\alpha) \chi_{\alpha}(x, u),$$

where  $\chi_{\alpha}(x, u) = \exp(2\pi i \alpha x) \exp(-2\pi i \{\alpha u\}_p)$  are characters of  $\Sigma_p$ ,  $\{\cdot\}_p$  is a fractional part of a  $p$ -adic number  $\{\cdot\}_p$  and

$$\widehat{f}(\alpha) = \int_0^1 \int_{\mathbb{Z}_p} f(x, u) \overline{\chi_{\alpha}(x, u)} dx d_p u$$

are Fourier coefficients. Hence Dirichlet kernels for  $\Sigma_p$  are

$$D_{m,n}(x, u) = \sum_{\alpha \in (-m, m) \cap p^{-n}\mathbb{Z}} \chi_{\alpha}(x, u), \quad m, n \in \mathbb{N}_0.$$

We proved in [2] that the Lebesgue constants have the asymptotics

$$L_{m,n} := \|D_{m,n}\|_{L_1(\Sigma_p)} = \int_0^1 \int_{\mathbb{Z}_p} |D_{m,n}(x, u)| dx d_p u \sim \frac{2}{\pi^2} \ln(m^2 p^n),$$

when  $m \rightarrow +\infty$ ,  $n \rightarrow +\infty$ . Consequently the Fourier series is divergent in  $L_1(\Sigma_p)$  and it is reasonable to consider Fejer kernels

$$F_{m,n}(x, u) = \sum_{\alpha \in (-m, m) \cap p^{-n}\mathbb{Z}} \left(1 - \frac{|\alpha|}{m}\right) \chi_{\alpha}(x, u), \quad m, n \in \mathbb{N}_0$$

that will be discussed in our talk and I will prove

**Theorem 1.** For all nonnegative integers  $n, m$   $\|F_{m,n}\|_{L_1(\Sigma_p)} = 1$ .

**References**

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2. Radyna A. Ya., Karpovich N.I. *The Lebesgue constants of  $p$ -adic solenoid* // *Vesnik Bielaruskaha Dziaržaŭnaha Universiteta*. Ser. 1. Math. 2012. No. 3. P. 87–90 (in Russian).

**SOLVING MATRIX DISCRETE THE FIRST ORDER EQUATIONS BY MEANS OF ALGEBRAIC MATRICIANT**

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Let  $K_0^{m \times m}$  be an algebra of matrix sequences with multiplication in the form of Laplace convolution. For matrix  $X = [x^{ij}]_{i,j=1}^m$  denote  $\tilde{m}_n(X) = \max_{1 \leq i,j \leq m} |x_n^{ij}|$ .

**Definition 1.** The sequence  $\tilde{m}(X) = \{\tilde{m}_0(X), \tilde{m}_1(X), \dots, \tilde{m}_n(X), \dots\}$  is called a majorizing sequence for matrix  $X \in K_0^{m \times m}$ .

**Definition 2.**  $X \in \ell_p^{m \times m}$ , when  $\forall i, j = \overline{1, m} \ x^{ij} \in \ell_p$ .

We define a norm of a matrix from  $\ell_p^{m \times m}$  by the following way  $\|X\|_{\ell_p^{m \times m}} = \|\tilde{m}(X)\|_{\ell_p}$ .  $\ell_p^{m \times m}$  is the Banach module under the Banach algebra  $\ell_1^{m \times m}$ .

Consider matrix algebraic homogeneous differential equation

$$DX = GX, \quad (1)$$

where  $D$  is the algebraic derivative operator (see [1]),  $G \in \ell_1^{m \times m}$ . A solution of the equation (1) is found in  $\ell_1^{m \times m}$  with initial condition  $X_0 = E$ . We use the method of successive approximations for building a solution of (1). The successive approximations are found from recursion relations

$$DX^{(n+1)} = GX^{(n)} \quad (2)$$

with initial approximation  $X^{(0)} = X_0 = E$ . Integrating (2) obtain successively

$$X^{(0)} = E, \quad X^{(1)} = E + \int G, \quad \dots, \quad X^{(k)} = E + \int G + \int G \int G + \dots + \underbrace{\int G \int G \dots \int G}_k, \quad \dots$$

**Definition 3.** The limit

$$\Omega^G = \lim_{k \rightarrow \infty} X^{(k)} = E + \int G + \int G \int G + \dots + \int G \int G \dots \int G + \dots,$$

when it exists, is called an algebraic matriciant of the equation (1).

Consider difference matrix homogeneous the first order equation

$$(n+1)X_{n+1} + (n\gamma + \delta)X_n = 0, \quad (3)$$

where  $\gamma, \delta \in \mathbb{C}^{m \times m}$ . A solution of the equation (3) is found in  $\ell_1^{m \times m}$  with arbitrary initial condition  $X_0$ . The equation (3) is transformed to algebraic differential equation (1), where  $G = (E - \gamma h)^{-1}(-\delta)$ ,  $h = \{0, 1, 0, \dots, 0, \dots\}$ . We obtain conditions for matrix  $\gamma$  under which  $\forall X_0$  there is a unique solution  $X = \Omega^G X_0$  of the equation (3), where  $\Omega^G$  is an algebraic matriciant of the equation (1). Corresponding to (3) inhomogeneous equation with arbitrary initial condition is investigated in a similar manner in the Banach module  $\ell_p^{m \times m}$ . Evaluations for solutions norms are obtained.

#### References

1. Васільєў І. Л., Навічкова Д. А. Матрычнае аднароднае рознаснае раўнанне першага парадку са зменнымі каэфіцыентамі ў камутатыўным выпадку // Вестн. БГУ. Сер. 1. 2014. № 1. С. 83–87.