

is a given function. Functions  $D(s)$  and  $K(s)$  were determined according to the following two models: 1) Brooks-and-Corey model; 2) Mualem-Van Genuchten model [1].

The numerical solution of above boundary-value problem was founded by meshfree radial basis function method [2].

Using the solutions of the above problem at the second part we found the minimum slope stability factor and possible slip surface using engineering methods such as Mozhevitinova, Fedorovsky — Kurylo, circular cylindrical sliding surfaces method, etc. [3], and optimization techniques for minimizing the objective function of many variables, such as coordinate descent method and golden section search.

As a result of these two steps it can be predicted what amount of moisture in the slope soil will lead to stability loss and dangerous geological processes such as landslides, avalanches, mudflows and so on.

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## ON SOME APPLICATIONS FOR EQUATIONS OF ELLIPTIC TYPE WITH SPECTRAL PARAMETER AND DISCONTINUOUS NONLINEARITY

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Models with discontinuous nonlinearities are used as idealization of continuous processes such that nonlinear parameters rapidly vary on small intervals in the range of phase variables. For many problems of hydrodynamics, thermophysics, electrophysics, control theory, mathematical biology, and other fields of modern natural science the mathematical models often include differential equations with discontinuous nonlinearities. As an examples, we can mention the Gol'dshtik's and the Lavrent'ev's problems for separated flows of incompressible fluid [1, 2]. Also, we can remember the Kuiper's problem on heating of conductor when voltage and temperature are constant on its surface, and electroconductivity of material changes jumpwise as a function of the temperature [3].

We consider the Gol'dshtik model for separated flows in incompressible fluid. Besides, a model problem approximately describing the flow of viscous incompressible fluid in a square cavern is solved. A solution of this two-dimensional problem of mathematical physics for a finite domain is found numerically using the Partial Differential Equation Toolbox of the MATLAB system by the finite element method. Estimations of differential operator for the problem are calculated. The existence of semiregular solutions of the Gol'dshtik problem is proved. Such solutions are of great interest in applied problems. The result on the number of solutions to the Gol'dshtik problem is gained by a variational method.

We study the Lavrent'ev mathematical model for separated flows with an external perturbation. This model consists of a differential equation with discontinuous nonlinearity

and the boundary condition. The coercive case is considered and the external perturbation is given in a concrete form. We consider the model of the external perturbation in the special analytical form that has not been studied for the Lavrent'ev problem. The existence of a semiregular solution is proved by the variational method. In addition, if variational functional has no more countable number of points of a global minimum, then there is the regular solution of the problem, i.e. the semiregular solution with the property of correctness. The regular solutions of the Lavrent'ev problem have not been investigated.

The problem on conductor heating is considered under constant voltage and constant temperature on the conductor surface when electroconductivity of material changes with a jump upon transition through certain temperatures. The results obtained earlier for problems with a spectral parameter for equations of elliptic type with discontinuous nonlinearities are applied to this problem of electrophysics. Restrictions on the gap points of nonlinearity are weakened (nonlinearity is electroconductivity of the conductor). The theorem on both the existence of the nonzero semiregular solution and the estimates for the differential operator of the problem under consideration is established.

We summarize the results of [1–3] for applications to equations of elliptic type with a spectral parameter and nonlinearity discontinuous with respect to the phase variable.

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## FINITE-DIFFERENCE ITERATIVE SOLVER WITH SPECTRALLY EQUIVALENT PRECONDITIONER FOR ANISOTROPIC ELECTRICAL IMPEDANCE TOMOGRAPHY PROBLEMS

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The electrical impedance tomography (EIT) problems in anisotropic inhomogeneous media like head tissues belongs to the class of the three-dimensional boundary value problems for elliptic equations with mixed derivatives. The efficiency of the most discussed and usable in practice numerical methods in context of modeling EIT problems is reviewed in [1,2], where it is shown that the best performance is demonstrated by the algebraic multi-grid (AMG) methods.

We present a novel type of the anisotropic bi-conjugate gradient iterative solver for 3D elliptic equations with mixed derivatives. The proposed numerical scheme is based on the finite-difference approximation of the problem in an arbitrary three-dimensional computational domain, augmented to a cuboid with non-conducting claddings and the Dirichlet type boundary conditions defined at the facets of the cuboid. The rectangular uniform finite difference grid is assumed to have high enough resolution across the characteristic layers of inhomogeneity. The nonconductive claddings imitate the Neumann