# BOUNDARY CONTROL IN DISTRIBUTED TRANSPORTATION NETWORKS

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In the paper the gas transportation model describing by the system of linear partial differential equations complicated by initial and boundary conditions is considered. The fundamental solution representation for the considered problem is based on exploiting the so-called canonical system formed by the collection of the eigenfunctions for the underlying operator and its adjoint that involve the initial and boundary data. This canonical system is the base of the multifunctional integral transformation for spatial variables which is the core of the developed operational calculus for PDE's. This approach was successfully applied in [1]. The state space parameters are gas pressure $p$ and mass flow $Q$ at the points of the pipe. All other physical parameters of the pipe and gas used here are constant at the moment of calculation. It is known that some important dynamic characteristics of the processes can be evaluated from the linearized model of the processes. The most accurate linear model can be realized in some neighborhood of the known basic regime $(\overline{Q}, \overline{p})$ of the considered process [2]. In particular, the following system of linear differential equations can be used for description of the disturbed state space parameters for the turbulent, isothermal gas flow in the unit pipeline

$$\frac{\partial Q(t,x)}{\partial t} = -s\frac{\partial p(t,x)}{\partial x} - \rho Q(t,x) - \beta p(t,x),$$
$$\frac{\partial p(t,x)}{\partial t} = \alpha\frac{\partial Q(t,x)}{\partial x}, \quad x \in [0,1], \quad t > 0.$$

where $x$ denotes the space variable, $t$ the time variable, $s$ the cross sectional area, $d$ the pipeline diameter, $c$ the isothermal speed of sound and $\lambda$ the friction factor, $\alpha, \beta, \gamma, \rho$ are some constants. The given equations can be rewritten in operator form as

$$D_t y(t,x) = Ly(t,x),$$

where

$$y(t,x) = \begin{bmatrix} y_1(t,x) \\ y_2(t,x) \end{bmatrix} \doteq \begin{bmatrix} Q(t,x) \\ p(t,x) \end{bmatrix}, \quad L = A + BD_x = \begin{bmatrix} -\rho & -sD_x - \beta \\ \alpha D_x & 0 \end{bmatrix}.$$

Here $D_t$ and $D_x$ describe the partial derivatives with respect to the time and space variables, respectively. The initial and boundary conditions are given as

$$y(0,x) = \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix} = \begin{bmatrix} u(x) \\ v(x) \end{bmatrix}, \quad \frac{\partial y_1(t,0)}{\partial x} = 0, \quad y_2(t,0) - y_2(t,1) = 0.$$

The initial control functions image the arisen disturbances $u(x), v(x)$ at $t = 0$ for the preassigned pumping regime $\overline{Q}(t,x), \overline{p}(t,x)$ in the pipeline, and the boundary data images the conservation of the gas pipeline pressure and pipeline storage in the inlet of pipe. Also, some other approximation can be introduced by exploiting the so-called $2D$ and repetitive models [3, 4].

**Refrences**

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## INVESTIGATION OF THE MOISTURE INFLUENCE ON SOIL MASS STABILITY ON THE SLOPE

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To determine the effect of moisture on the stability of the soil mass, we divide the problem into two parts: 1) determination the saturation change in soil due to pipeline damage, and 2) optimization problem of searching the minimum stability factor and the critical slip surface.

First we considered two-dimensional problem of soil slope saturation (domain $\Omega$) as a result of the pipeline damage (point $D$, $D \in \partial\Omega$, where $\partial\Omega$ is a border of $\Omega$). Therefore it is obtained two subdomains of complete and incomplete saturated soil. Our task is to determine the dynamics of saturation region with time.

For this purpose Richards' equation was used [1] with regard to saturation function $s(x,z,t) = (\theta - \theta_{\min})/(\theta_{\max} - \theta_{\min})$, where $\theta_{\min}$ is the residual liquid content in the porous medium; $\theta_{\max}$ is the saturated liquid content in a porous medium; $0 \leqslant \theta_{\min} \leqslant \theta \leqslant \theta_{\max} = \sigma$, $\sigma$ is a porosity and $\theta$ is the volumetric liquid content in the unit volume of a soil.

Then the mathematical model of the investigated process can be written by the following boundary-value problem:

$$\frac{\partial s(x,z,t)}{\partial t} = \frac{\partial}{\partial x}\left(D(s)\frac{\partial s(x,z,t)}{\partial x}\right) + \frac{\partial}{\partial z}\left(D(s)\frac{\partial s(x,z,t)}{\partial z}\right) - \frac{1}{\theta_{\max} - \theta_{\min}} \cdot \frac{\partial K(s)}{\partial z},$$

$$(x,z) \in \Omega, \quad t \in (0;T];$$

$$s(x,z,0) = s_0(x,z), \quad (x,z) \in \overline{\Omega} = \Omega \cup \Gamma;$$

$$s(x,z,t)|_{\Gamma_1} = s_1(x,z,t), \quad (x,z) \in \Gamma_1; \quad \left(-D(s)\frac{\partial s(x,z,t)}{\partial z}\right)\Bigg|_{\Gamma_2} = 0, \quad (x,z) \in \Gamma_2, \quad t > 0;$$

$$\frac{\partial s(x,z,t)}{\partial x}\Bigg|_{\Gamma_3 \setminus \{D\}} = 0, \quad (x,z) \in \Gamma_3 \setminus \{D\}; \quad s(x,z,t)|_D = 1, \quad (x,z) \in D, \quad t > 0,$$

where $D(s)$ is the soil liquid diffusivity that can be treated as a nonlinear diffusion coefficient; $K(s)$ is a hydraulic conductivity of an unsaturated porous medium; $s_1(x,z,t)$