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ORDINARY DIFFERENTIAL EQUATIONS

On the Global Lyapunov Reducibility of Two-Dimensional Linear Systems with Locally Integrable Coefficients

A. A. Kozlov and I. V. Ints
Polotsk State University, Polotsk, Belarus
e-mail: kozlovaa@tut.by, i.ints@mail.ru

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Abstract—We show that if a two-dimensional linear nonstationary control system with locally integrable and integrally bounded coefficients is uniformly completely controllable, then the corresponding linear differential system closed with a measurable bounded control linear in the state variables has the property of global Lyapunov reducibility.

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Consider the linear nonstationary control system

$$\dot{x} = A(t)x + B(t)u, \qquad x \in \mathbb{R}^n, \qquad u \in \mathbb{R}^m, \qquad t \geq 0, \tag{1}$$

with coefficient matrices A and B that are Lebesgue integrable and integrally bounded [1, p. 252]. By closing system (1) with a control u defined in the form of the linear feedback

$$u = U(t)x, \tag{2}$$

where U is some measurable bounded $m \times n$ matrix, we obtain the closed-loop system

$$\dot{x} = (A(t) + B(t)U(t))x, \qquad x \in \mathbb{R}^n, \qquad t \geq 0, \tag{3}$$

whose coefficients are locally integrable and integrally bounded as well. The problem of constructing a control (2) for system (1) ensuring the asymptotic (Bogdanov) [2] equivalence of system (3) closed by this control and an arbitrary given linear system

$$\dot{z} = C(t)z, \qquad z \in \mathbb{R}^n, \qquad t \geq 0, \tag{4}$$

with a measurable integrally bounded coefficient matrix C (i.e., to provide the existence of a Lyapunov transformation [3, pp. 153–154] relating systems (3) and (4)) is called the *problem of global Lyapunov reducibility of the linear system* (3) [4; 5, pp. 258–259]. In this case, all Lyapunov invariants [5, pp. 46–80] of system (3) with the control U and these of system (4) coincide, and that is why the problem of global Lyapunov reducibility is sometimes called the *problem of global control of the full set of Lyapunov invariants* [4]. It is well known (e.g., see [5, p. 31; 6–10]) that the Lyapunov characteristic exponents are asymptotic invariants of linear systems; therefore, the problem stated above is a natural generalization of the problem of global control of the Lyapunov exponents of linear nonstationary systems [7, p. 41; 8, pp. 303–305; 9; 10]. In turn, the latter is a straightforward generalization of the well-known spectrum assignment problem [6, pp. 78–80, 141 of the Russian translation] to the case of nonstationary systems (1).

The global Lyapunov reducibility of system (3) is studied under the assumption of uniform complete controllability of the original system (1).

Definition 1 [10, 11]. System (1) is said to be *uniformly completely controllable* if there exist numbers $\sigma > 0$ and $\gamma > 0$ such that, for any $t_0 \geq 0$ and $x_0 \in \mathbb{R}^n$, there exists a measurable bounded

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Author information

Affiliations

1. **Polotsk State University, Polotsk, Belarus**
   A. A. Kozlov & I. V. Ints

Corresponding author

Correspondence to A. A. Kozlov.

Additional information

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