Application of Harmonic Wavelets to Processing Oscillating Hydroacoustic Signals

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Abstract— The paper is devoted to the application of specific functions called harmonic wavelets, which are aimed at processing a wide range of oscillating hydroacoustic signals including multiharmonic and transient signals. We provide basics of the harmonic wavelet transform and a two-stage algorithm for computing wavelet coefficients based on the discrete Fourier transform. We introduce a special efficiency factor of applying these wavelets to oscillating hydroacoustic signals.

Application of harmonic wavelets is efficient for processing oscillating hydroacoustic signals since harmonic wavelets have similarities with these types of signals. In many cases the best basis is the basis that has high correlation with the investigated signals since signal representation in such a basis will require a small number of components.

We devote special attention to a very important practical task — denoising of oscillating signals using special statistical criteria and wavelet-based thresholding.

1. INTRODUCTION.

Hydroacoustic signal processing [1–3] is performed using different techniques: filtering in the time and frequency domains [4–8], finding correlations and statistical connections, non-linear information processing, etc. In most cases several different techniques are used in reception paths because it allows us to achieve the best results for wanted signal extraction and classification. Most reception paths are complex analogue or digital and analogue devices aimed at these tasks and we normally have restrictions on the processing time.

In modern reception paths of hydroacoustic means we widely apply digital signal processing techniques. Achievements in computational device development enable us to use the advantages of digital fast processing of hydroacoustic information using principles of processing radar signals.

In practice we need to make an appropriate choice of techniques and equipment for hydroacoustic signal processing. In several cases signal processing is aimed at specific signals, e.g., phase shift-keyed or frequency shift-keyed signals. Sometimes we need to resist reverberation disturbance. Researchers also use logical methods of selecting and estimating the informativity of different features that are included in the original feature space for objects.

Thus, optimal processing of hydroacoustic information consists of elementary operations represented mathematically. Optimal devices are used for each operation and then we perform joint optimization of a set of decision-making devices. For example, we can perform optimization for signal envelope using the Hilbert transform or polynomial smoothing, optimization for signal filtering using matched filters maximizing signal-to-noise ratio.

Researchers tend to select signal processing algorithms and techniques in order to match signals under study. For example, we can apply harmonic wavelets for oscillating signal processing including multiharmonic signals.

The paper is devoted to the use of specific functions called harmonic wavelets and aimed at processing a wide class of oscillating hydroacoustic signals (processes), in particular, multiharmonic processes. The use of harmonic wavelets for oscillating signal processing is particularly efficient because harmonic wavelets are similar to these types of signal. In many cases we search for a basis with high correlation with signals under study because signal representation in such a basis requires a small number of components. We bring particular attention to a very important practical task of signal preprocessing — signal denoising using special statistical criteria and wavelet thresholding.

The contents of the paper can be represented the following way:

- we introduce harmonic wavelets;

- we illustrate harmonic signal analysis together with noise using harmonic wavelets;
- we illustrate signal denoising using harmonic wavelets and special wavelet based thresholding. We provide formulas for threshold computation for the harmonic wavelet transform. We obtain analytical expressions for modified thresholds in order to apply wavelet-based thresholding;
- we illustrate the application of the suggested denoising technique to a multiharmonic hydroacoustic signal.

2. HARMONIC WAVELETS

Consider a basis based on wavelets with spectra in the form of a rectangular wave in the given frequency band [9, 10]. For example, at the zero level ($j = 0$) for the basis wavelet we have the following spectral density expression [11] $W(\omega)$, taking phase equal to zero:

$$W(\omega) = \begin{cases} 1/2\pi, & 2\pi \leq \omega < 4\pi \\ 0, & \omega < 2\pi, \omega \geq 4\pi. \end{cases} \tag{1}$$

Then we can find the form of the basis function in the time domain, using the inverse Fourier transform of expression (1) ($i = \sqrt{-1}$):

$$w(x) = \frac{e^{i4\pi x} - e^{i2\pi x}}{i2\pi x} \tag{2}$$

Complex representation of the wavelet allows us to obtain two real representations on the basis of a single representation. It is similar to the way of using function $e^{i\omega x}$ in complex harmonic analysis to represent two real functions in a sinusoidal $\sin(\omega x)$ and cosine $\cos(\omega x)$ forms (based on the Euler formula). Obviously the real part is even and the imaginary part is an odd function.

Consider the expression for the spectrum of the basis wavelet for j-th decomposition level and the shift by an amount k:

$$W(\omega) = \begin{cases} \frac{1}{2\pi} 2^{-j} e^{-\frac{i\omega k}{2^j}}, & 2\pi 2^j \leq \omega < 4\pi 2^j \\ 0, & \omega < 2\pi 2^j, \omega \geq 4\pi 2^j, \end{cases} \tag{3}$$

Similarly, we will find the inverse Fourier transform from (3)

$$w(2^j x - k) = \frac{e^{i4\pi(2^j x - k)} - e^{i2\pi(2^j x - k)}}{i2\pi(2^j x - k)}, \tag{4}$$

where $j \geq 0$, $|k| < \infty$. Harmonic wavelets have finite (compact) support in the frequency domain and infinite support in the time domain (since the basis wavelet spectrum is different from zero in a limited frequency band). However, since in the time domain the wavelet decreases according to the hyperbolic law, we can determine the effective width of the support in the time domain.

If $j = -1$ (this corresponds to the scaling function) $W(\omega)$ has the following form [9]:

$$W(\omega) = \begin{cases} \frac{1}{2\pi} e^{-i\omega k}, & 0 \leq \omega < 2\pi \\ 0, & \omega < 0, \omega \geq 2\pi, \end{cases} \tag{5}$$

In the time domain we obtain the following expression:

$$\phi(x - k) = \frac{e^{i2\pi(x-k)} - 1}{i2\pi(x - k)}, \tag{6}$$

where $|k| < \infty$, $\phi(x)$ is the scaling function. The support of this function can be represented in a way similar to the support of the wavelet. The reason for selecting such a scaling function (6) and basis wavelet (4) is that they form an orthogonal set:

$$\int_{-\infty}^{+\infty} w(2^j x - k) w(2^r x - s) dx = 0 \quad \forall j, k, r, s \ (j, r \geq 0) \tag{7}$$

$$\int_{-\infty}^{+\infty} w(2^j x - k) w^*(2^r x - s) dx = 0 \quad \forall j, k, r, s \ (j, r \geq 0; \ r \neq j; \ s \neq k) \tag{8}$$

$$\int_{-\infty}^{+\infty} |w(2^j x - k)|^2 dx = 1/2^j \tag{9}$$

Now we can formulate the following properties of harmonic wavelets which make them similar to other classes of wavelets:

- Harmonic wavelets have compact support in the frequency domain, which can be used for localizing signal features (in the frequency domain);
- There are fast algorithms for computing wavelet coefficients and reconstructing signals in the time domain. These algorithms are based on the fast Fourier transform (FFT).

The drawback of harmonic wavelets are their weak localization properties in the time domain in comparison with other types of wavelets. The spectrum in the form of a rectangular wave leads to the decay in the time domain as $1/x$, which is not sufficient for extracting short-term singularities in a signal in the time domain.

3. SIGNAL DENOISING BASED ON HARMONIC WAVELETS AND THRESHOLDING

Consider the problem of signal denoising using multiharmonic signals. Harmonic wavelets are suitable for such signals and make it possible to perform multiscale analysis [12].

Signal denoising will be performed using thresholding of wavelet coefficients [12, 13]. Thresholding is one of the most accurate techniques of signal denoising together with wavelets. Thresholding allows us to avoid problems that can arise in the case of binary classification of wavelet coefficients when we classify these wavelet coefficients into those that contain noise components and those that contain signal information. The estimate of standard deviation is based on statistical estimates and thresholding is not applied to all decomposition levels, but to initial ones (noise levels). Then we perform reconstruction of denoised signal using the modified wavelet coefficients. We also use a special criterion for determining the levels for thresholding.

First, we will represent wavelet coefficients on noise levels in the following form (due to linearity of the harmonic wavelet transform):

$$a_j(k) = w_j(k) + e_j(k) \tag{10}$$

where j is the number of decomposition level, $w_j(k)$ are wavelet coefficients of the denoised signal (when there is no noise), $e_j(k)$ are wavelet coefficients of noise. In this case the component $w_j(k)$ can be determined the following way:

$$w_j(k) = F\left[a_j(k), \rho_j\right] \tag{11}$$

where F is the thresholding operator, ρ_j is the threshold value for denoising. Formula (11) allows us to distinguish between soft and hard thresholding [12], which can be written the following way:

1. Hard thresholding of wavelet coefficients:

$$w_j(k) = \begin{cases} a_j(k), & |a_j(k)| > \rho_j \\ 0, & |a_j(k)| \leq \rho_j. \end{cases} \tag{12}$$

2. Soft thresholding of wavelet coefficients:

$$w_j(k) = \begin{cases} a_j(k) - \rho_j, & a_j(k) > \rho_j \\ 0, & -\rho_j < \ a_j(k) \leq \rho_j \\ a_j(k) + \rho_j, & a_j(k) \leq -\rho_j. \end{cases} \tag{13}$$

The key role in the thresholding is played by thresholds ρ_j which can be calculated the as (multilevel thresholds):

$$\rho_j = \sigma_j \sqrt{2 \ln N} \tag{14}$$

where σ_j is the estimate of standard deviation of the noise component at the jth decomposition level. This expression means [12, 13] that wavelet coefficients of the Gaussian noise with the standard deviation σ_j will not exceed the threshold ρ_j with probability P tending to 1 when $N \to \infty$. Wavelet coefficients corresponding to the samples of noise will be equal to zero. The computation of standard deviation is performed for the finest decomposition level and then we perform the computation of standard deviation for other levels. The estimate of standard deviation σ_f for the finest level can be performed by the following formula [12, 14]:

$$\sigma_f = \frac{\text{median} \left| a_j(k) - \text{median} \left\{ a_j(k) \right\} \right|}{0.6745} \tag{15}$$

This estimate is robust [12, 14] due to the robustness of the median and this estimate is stable to signal outliers (these outliers can be caused by short failures in the equipment). This estimate is obtained [14] under the assumption of the Gaussian distribution of noise and the Gaussian model used in this paper is one of the most widespread in practice. Thus, the formula for the finest decomposition level has the following form:

$$\rho_f = \sigma_f \sqrt{2 \ln N} \tag{16}$$

For other levels the formula is:

$$\rho_j = \frac{\sigma_f}{q(j)} \sqrt{2 \ln N}, \tag{17}$$

where $q(j)$ is the normalized coefficient depending on a decomposition level. Next, we need to find this normalized coefficient. It can be found using the following procedure. When we process signals with white Gaussian noise (with the given standard deviation) by a set of octave filters with the magnitude response equal to 1 in the passband, the standard deviation of the next level will be decreased twice in comparison with the previous level:

$$\sigma_j = \frac{\sigma_f}{2^j}. \tag{18}$$

Unlike the standard thresholding (in the case of real signals) in this case thresholding is performed separately for the real and imaginary part and then we compute the inverse wavelet transform from modified wavelet coefficients for returning in the time domain. For any complex-valued sequence x the following expression is valid:

$$\sigma_x^2 = \sigma_{real(x)}^2 + \sigma_{imag(x)}^2, \tag{19}$$

where $\sigma_{real(x)}^2$ is the variance of the real part of sequence x, $\sigma_{imag(x)}^2$ is the variance of the imaginary part of sequence x. In our case the variance of complex wavelet coefficients is determined using variances of real and imaginary parts. Considering $\sigma_{real(x)}^2 \approx \sigma_{imag(x)}^2$ (since real and imaginary parts of the kernel of the harmonic wavelet transform can be considered equal and statistically independent, which can be confirmed by experiments), we arrive at

$$\rho_j = \frac{\sigma_f}{\sqrt{2} \cdot 2^j} \sqrt{2 \ln N} \tag{20}$$

where ρ_j is the threshold applied to real and imaginary parts of wavelet coefficients, σ_f is estimated using complex wavelet coefficients.

We are going to simulate the multiharmonic hydroacoustic signal with the alternation of harmonics with different amplitudes and frequencies: $f_1 = 10\,\text{Hz}$, $f_2 = 15\,\text{Hz}$, $A_1 = 5$, $A_2 = 10$ ($f_s = 1024\,\text{Hz}$, $N = 4096$). The signal has additive noise (the noise is considered to be white with Gaussian distribution) with zero mean and standard deviation $\sigma = 0.8$. Consider thresholding using the denoised signal shown in Fig. 1.

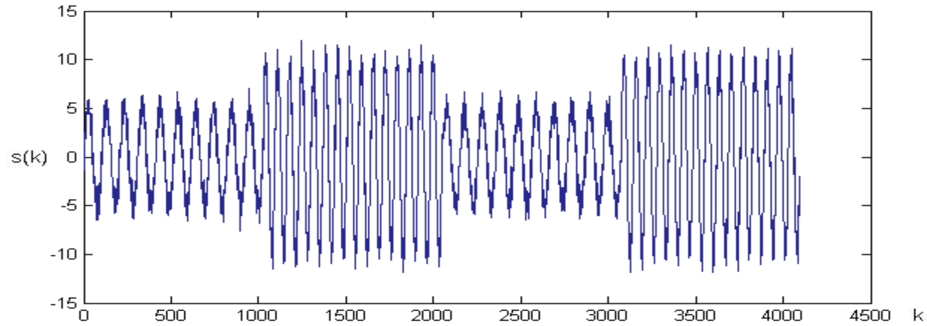

Figure 1: Denoised multiharmonic signal with alternation of harmonics.

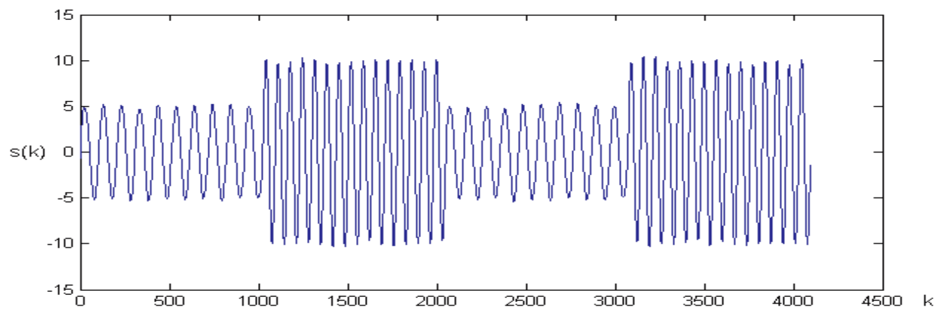

Figure 2: Denoised signal using soft thresholding.

We will next apply soft thresholding since the original signal has no isolated singularities (local outliers, etc.). As stated earlier [12, 13, 15] thresholding can be applied to 4 initial decomposition levels. The plot of the denoised signal is shown in Fig. 2:

There are several approaches allowing us to estimate the quality of signal denoising:

1) Estimate of the standard deviation of noise compared to the original value of the standard deviation.

2) Statistical criteria for checking the whiteness of noise such as the criterion based on the autocorrelation function.

Table 1: Estimation of the quality of signal denoising.

True value of the standard deviation of noise	Estimation of the standard deviation of the extracted noise	$\gamma_j(\%)$
0.8	**0.7951**	**1.62**

The results provided in Table 1 show high quality of signal denoising.

4. CONCLUSION

We have considered a specific class of wavelets — harmonic wavelets and demonstrated signal denoising using harmonic wavelets, statistical criteria and a specialized thresholding procedure. We have also provided the modification of thresholds for harmonic wavelets. We have obtained good results for processing a multiharmonic hydroacoustic signal with alternation of harmonics, which can be confirmed by data in Table 1.

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REFERENCES

1. Hodges, R. P., *Underwater Acoustics: Analysis, Design, and Performance of Sonar*, 353, Wiley Interscience, UK, 2010.

2. Kaplun, D., D. Klionskiy, A. Voznesenskiy, and V. Gulvanskiy, "Digital filter bank implementation in hydroacoustic monitoring tasks," *Przeglad Electrotechniczny (Electrical Review)*, Vol. 91, No. 2, 47–50, 2015.
3. Milne, P. H., *Underwater Acoustic Positioning Systems*, Gulf Publishing Co., 1983.
4. Kaplun, D. I., D. M. Klionskiy, A. S. Voznesenskiy, and V. V. Gulvanskiy, "Application of polyphase filter banks to wideband monitoring tasks," *IEEE NW Russia Young Researchers in Electrical and Electronic Engineering Conference (ElConRusNW)*, 97–100, Russia, Saint Petersburg, 2014.
5. Kaplun, D. I., D. M. Klionskiy, A. S. Voznesenskiy, and V. V. Gulvanskiy, "Order reduction of a low-frequency prototype filter by its magnitude response symmetrization in the task of filter bank design for wideband monitoring," *International Scientific and Technical Conference Dedicated to the 50th Anniversary of MREI-BSUIR*, 270–271, Belarus, Minsk, 2014.
6. Crochiere, R. E. and L. R. Rabiner, *Multirate Digital Signal Processing*, 411, Prentice Hall, USA, 1983.
7. Mitra, S. K., *Digital Signal Processing: A Computer-based Approach*, 940, McGraw-Hill, USA, 1998.
8. Antoniou, A., *Digital Filters: Analysis, Design, and Applications*, 689, McGraw-Hill, USA, 1993.
9. Newland, D. E., "Harmonic wavelet analysis," *Proceedings of the Royal Society of London, Series A (Mathematical and Physical Sciences)*, Vol. 443, No. 1917, 203–225, Oct. 8, 1993.
10. Newland D. E.,*Random Vibrations, Spectral and Wavelet Analysis*, 3rd Edition, Longman, Harlow, and John Wiley, New York, 1993.
11. Antoniou, A., *Digital Signal Processing: Signals, Systems, and Filters*, 864 pages, McGraw-Hill, 2006.
12. Mallat, S., *A Wavelet Tour of Signal Processing*, 2nd Edition, Academic Press, 1999, ISBN 0-12-466606-X.
13. Donoho, D. L. and J. M. Johnstone, "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, Vol. 81, No. 3, 425–455, 1994.
14. Huber, P. J., *Robust Statistics*, 308 pages, John Wiley & Sons, NY, 1981.
15. Box, G. E. P. and G. M. Jenkins, *Time Series Analysis: Forecasting and Control*, Holden Day, San Francisco, 1990.

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