Correlation Search for Binary Objects in Images by Factorizing Raster Matrices

S. V. Ablameyko*, R. P. Bogush, and S. V. Mal'tsev****

**Institute of Engineering Cybernetics, National Academy of Sciences of Belarus, ul. Surganova 6, Minsk, 220012 Belarus*
e-mail: abl@newman.bas-net.by
***Polotsk State University, ul. Blokhina 29, Novopolotsk, 211440 Belarus*
e-mail: bogush@psu.unibel.by

Abstract—A method of correlation search for binary objects in images by factorizing raster matrices is proposed. A mathematical apparatus for factorizing raster binary matrices is considered. An expression for the upper bound of computational complexity of a 2D correlation function is obtained. An algorithm for detecting objects rotated by 180° relative to the horizontal axis without additional computational costs is developed. The results of experiments are presented.

1. INTRODUCTION

A search for objects in images is one of the key problems in visual data processing applications [1–3]. Conventional procedures for detecting objects employ correlation algorithms realizing all the advantages of the maximum-likelihood method. A correlation function then characterizes the degree of matching of the objects compared [3].

One of the main methods for detecting objects is the template matching method [4, 5]. All the objects in the image are compared to the template by scanning the image, usually from left to right and from top to bottom. The estimate is based on mutual correlation of the input and template images [6].

Formally, the correlation processing of images consists in calculating a 2D vector–matrix product (VMP) and analyzing the values of the correlation matrix obtained. However, the correlation methods are sensitive to the object's rotation and scaling, which requires a large number of templates and increases the computational burden of determining the correlation functions. To overcome this shortcoming, one can use various 2D transforms which substantially reduce the number of multiplications and additions necessary for calculating the correlation function [7, 8]. The diminishing of the computational costs is attained by synthesizing fast algorithms for digital processing based on expansion (factorization) of the original matrix in a series of sparse matrices. Such an approach is possible for the major part of the existing algorithms employing double transforms [7, 8].

In many systems, an image is represented in a binary (black-and-white) format. In processing of binary images, the use of the above transforms leads to additional computational costs caused by application of complex arithmetics, arithmetics of digital rings, or alternative mathematical structures. Therefore, the correlation functions of binary images can be calculated more effectively by direct approaches based on factorization of the raster matrices of the template images [1, 9, 10].

In this work, we present a method of the correlation search for images employing factorization of the raster binary matrices. This method does not require additional additive calculation of the elements of the correlation matrix and enables one to detect an object identical to the template and rotated by 180° relative to the horizontal axis. We also present the results of experiments.

2. SEARCH FOR THE OBJECTS IN AN IMAGE BY A DIRECT APPROACH

Let us introduce definitions.

The pth leading diagonal of a rectangular matrix $[A] = \{a_{ij}\}$, $i = \overline{1, M}$, $j = \overline{1, N}$, $M < N$, and $p = (0, \ldots, N - M)$ is the diagonal formed by the elements $(a_{i, j = i + p})$.

The pth secondary diagonal of a rectangular matrix $[A] = \{a_{ij}\}$, $i = \overline{1, M}$, $j = \overline{1, N}$, $M < N$, and $p = (0, \ldots, N - M)$ is the diagonal formed by the elements $(a_{M - i + 1, j = i + p})$.

Consider a problem of the correlative search for a given object $A = [a_{ij}]$ of size $n \times n$ in an image $D = \{d_{ij}\}$ of size $N \times N$. An elementwise comparison of the pixels of the object and the image involves an extraction of the group of elements in the image of the size $N \times n$. Then, the search for an object A in the extracted block $D_1 = \{d_{ij}\}$, $i = \overline{1, N}$, and $j = \overline{1, n}$ is reduced to determining the correlation of the rows of the object and the image. Mathematically, such a procedure repre-

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sents a multiplication of the transposed matrix of the extracted block by the matrix of the object:

$$H = A \times D_1^T,$$

where $[H]$ is the matrix of the correlation coefficients of the rows in the extracted fragment and the object.

The aim is the detection of the object as a whole. Therefore, an object A is detected in the extracted block D_1 if n rows in $[A]$ sequentially coincide with n rows in $[D_1]$. The above condition is satisfied if the maximum correlation coefficients are obtained. These coefficients must lie at one of the leading diagonals of the matrix $[H]$, which formally follows from the definition of the product of two matrices. The starting position of A in D_1 depends on the position of the first element of the pth diagonal.

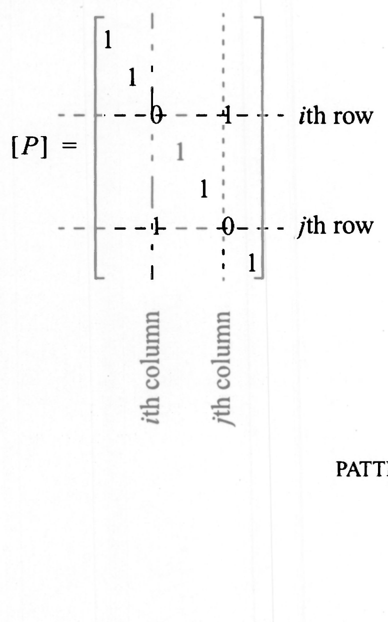
The selection of the pth diagonal with maximum elements in the matrix H is equivalent to analyzing the elements of the vector $[\bar{X}]$ represented as

$$\bar{X}_p = \sum_{i=1}^{n} h_{i,\, j=i+p}. \quad (1)$$

A decision regarding the presence of the object A in the pth zone of the block D_1 is made by comparing the elements of the vector $[\bar{X}]$ with a threshold level related to the signal-to-noise ratio in the image D.

In practical applications, it is often necessary to search for an object rotated by 180° relative to the horizontal axis. In matrix representation, a rotation of the object A by 180° relative to the horizontal axis is equivalent to a simple permutation of rows. For an object of size $m \times n$ occupying the rows ranging from i to k in an image of size $M \times N$, the rotation is equivalent to substituting the kth, $(k-1)$th, $(k-2)$, etc., rows for the ith, $(i-1)$th, $(i-2)$, etc., rows, respectively. If $m \neq 0$, the row with the number $(i+k)/2$ remains unchanged.

In the matrix form, the transposition of a row from the ith position to the jth one can be done by multiplying on the left the original matrix by a permutation matrix given by



Hence, the correlation matrix H_1^R of the object A and a fragment D_1 of the image is calculated with allowance for the rotation of the object by 180° relative to the horizontal axis as

$$[H_1^R] = [A(P_1P_2\ldots P_mD_1)^T].$$

It is known [11] that $[FG]^T = [G]^T[F]^T$; therefore,

$$[H_1^R] = [AD_1^TP_1^TP_2^T\ldots P_{m/2}^T]$$
$$= [H_1P_1^TP_2^T\ldots P_{m/2}^T].$$

Hence, it is possible to detect the rotated object by analyzing the correlation coefficients lying at the pth secondary diagonals of the matrix $[H_1]$ based on the analysis of the elements of the vector $[\bar{Y}]$ represented as

$$\bar{Y}_p = \sum_{i=m}^{1} h_{i,\, j=i+p}. \quad (2)$$

The algorithm of searching for the objects based on direct matrix multiplication involves the following steps.

(i) Extraction of the first fragment $D_1 = d_{ij}$ of size $M \times n$, $i = \overline{1, M}$ and $j = \overline{1, n}$ in the left-hand side of the raster image.

(ii) Multiplication of the matrix of the extracted fragment D_1^T by the matrix of the template A.

For calculating the correlation functions of the two raster binary matrices by VMP, we take into account that 0 and 1 are inverse symbols in terms of the raster binary data and introduce the following conditions:

$$0 \times 0 = 1, \quad 1 \times 0 = -1, \quad 0 \times 1 = -1, \quad 1 \times 1 = 1. \quad (3)$$

(iii) Analysis of the values of the elements in the obtained correlation matrix $[H] = \{h_{i,j}\}$ using expressions (1) and (2).

(iv) A shift to the right by one element in the image, selection of the next fragment of the size $n \times M$ and transfer to step (ii) if the number of shifts is less than $(N-n)$. In the opposite case, the procedure is terminated. The total number of the fragments analyzed is $(N-n+1)$.

Example.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

$$D = \left[\begin{array}{cccccc|cccccccccccc}
0&0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0\\
1&1&1&1&1&1&0&0&1&1&0&0&0&1&1&0&0&0\\
0&1&1&1&1&0&0&1&1&1&1&0&0&1&1&0&0&0\\
0&0&1&1&0&0&0&0&1&1&0&0&1&1&1&1&0&0\\
0&1&1&1&1&0&0&0&1&1&0&0&0&1&1&0&0&0\\
0&0&1&1&0&0&0&0&0&0&0&0&1&1&1&1&0&0\\
0&0&1&1&0&0&0&0&0&0&0&1&1&1&1&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0&0\\
0&0&1&1&0&0&0&1&1&1&1&0&0&0&0&0&0&0\\
0&0&1&1&0&0&0&0&1&1&0&0&0&0&0&0&0&0\\
0&1&1&1&1&0&0&1&1&1&1&0&0&0&1&1&0&0\\
0&0&1&1&0&0&0&0&1&1&0&0&0&0&1&1&0&0\\
0&1&1&1&1&0&0&0&1&1&0&0&0&1&1&1&1&0\\
1&1&1&1&1&1&0&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1
\end{array}\right].$$

Based on the algorithm proposed, the search for object A in image D involves the analysis of the elements of correlation matrix H obtained by multiplying a raster image of object A by fragments $D_1, \dots, D_{13}$ of the image analyzed. For fragment D_1, correlation matrix $[H_0]$ of size 6×18 is written as

$$[H_0^T] = \begin{bmatrix}
-6&-6&\mathbf{6}&2&-2&2&-2&-2&-6&-6&-2&-2&2&-2&2&\mathbf{6}&-6&-6\\
-2&-2&2&\mathbf{6}&2&6&2&2&-2&-2&2&2&6&2&\mathbf{6}&2&-2&-2\\
2&2&-2&2&\mathbf{6}&2&6&6&2&2&6&6&2&\mathbf{6}&2&-2&2&2\\
-2&-2&2&6&2&\mathbf{6}&2&2&-2&-2&2&2&\mathbf{6}&2&6&2&-2&-2\\
2&2&-2&2&6&2&6&6&2&2&6&\mathbf{6}&2&6&2&-2&2&2\\
2&2&-2&2&6&2&6&\mathbf{6}&2&2&6&6&2&6&2&-2&2&2
\end{bmatrix}^T.$$

Then vectors $\bar{X}$ and $\bar{Y}$ corresponding to matrix $[H_0]$ are represented as

$$\bar{X} = [4, 8, 36, 16, 16, 16, 12, 8, 8, 16, 4, 16, 12].$$

$$\bar{Y} = [12, 16, 4, 16, 8, 8, 12, 16, 16, 16, 36, 8, 4].$$

The analysis of the correlation functions shows that the object identical to the template and the object rotated relative to the horizontal axis start from the third and 11th rows of the fragment processed, respectively.

To reduce the number of necessary operations, we use the following property: the calculation of a 2D correlation can be represented as a series of calculations of one-dimensional correlation functions of the image and template rows, i.e., as a VMP calculation. In this case, the computational costs of multiplying a vector by a binary matrix can be decreased by expanding (factorizing) the original (template) matrix in a series with respect to sparse matrices, whose rows contain no more than two data elements, and a sequential multiplication of the vector formed by each row of the processed fragment by each matrix of the expansion.

3. BASIC NOTIONS OF FACTORIZATION

Let us introduce definitions necessary for factorizing raster binary matrices.

A binary image matrix or raster binary matrix is a matrix $[A]$ of size $M \times N$ with the elements $a_{ij} = 1, 0$, $i = \overline{1, M}$ and $j = \overline{1, N}$.

Zero and unity are the information elements (symbols) of the raster binary matrix $[A]$.

The elements of the ith row of raster binary matrix $a_{ij} = 0(1)$ and $a_{ik} = 1(0)$ are called inverse (opposite). The inversion is treated in terms of representation of the raster binary data. Then, row a_i is inverse to row a_k if all the elements of the former are opposite to the elements of the latter.

Each row of a sparse raster binary matrix $[D]$ contains at least one information symbol; other positions are vacant. There is no notation for the vacant positions of matrix d_{ij}. The absence of the information symbol allows elimination of the corresponding position from processing.

For the sparse matrices, row a_i is inverse to row a_k if all the information symbols of the former are opposite to those of the latter.

Let us introduce the rules for multiplying elements of the sparse raster binary matrices with allowance for the alphabet proposed:

$$0 \times 0 = 1, \quad 0 \times 1 = 0, \quad 1 \times 0 = 0, \quad 1 \times 1 = 1.$$

The absence of the information symbol in position (ij) allows skipping the multiplication of this element by the element whose position is determined by the rules of VMP calculation. Such a position is not taken into account in multiplying sparse raster binary matrices.

Statement 1. Any raster matrix $[A]$ of the size $M \times N$ containing zeros and unities can be represented as a product

$$[A] = [D] \times [B],$$

where B is a matrix with size $N_1 \times N$, $N_1 \le M$ derived from matrix $[A]$ by eliminating repeating and inverse rows and D is a matrix of size $M \times N_1$ containing the following elements:

$$d_{ij} = \begin{cases} 1 \text{ if } b_j = a_i \\ 0 \text{ if } b_j = -a_i \\ \text{otherwise, the information symbol is absent.} \end{cases} \quad (4)$$

Proof. The validity of the statement is evident, since matrix $[D]$ represents a combination of the permutation matrices, which position the rows of matrix $[B]$ in matrix $[A]$ with inversions or repetitions. Each row of $[D]$ matrix contains a single information symbol. The multiplication of the sparse matrices makes it possible to prove the validity of the first statement:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \\ & 1 \\ & 0 \\ & 1 \\ 1 & \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Statement 2. Any raster matrix $[A]$ of size $M \times N$ containing zeros and unities can be represented as a product of h sparse raster matrices with block–diagonal structure, each of which contains no more than two information symbols in a row ($h = \lfloor \log_2 N \rfloor + 1$, where $\lfloor * \rfloor$ stands for the nearest integer).

Proof. Let us represent matrix $[A]$ of size $M \times N$ as

$$[A] = \lfloor A_1 \vdots A_2 \vdots \ldots \vdots A_{p+1} \rfloor, \quad (5)$$

where $[A_j]$, $\{j = \overline{1, p}\}$ is a matrix of size $M \times k$; k is an arbitrary integer; and $[A_{p+1}]$ is a matrix of size $M \times (N - kp)$. Using the first statement, one can represent matrix $[A^T]$ from expression (5) as

$$[A^T] = \begin{bmatrix} A_1^T \\ \ldots \\ A_{p+1}^T \end{bmatrix} \begin{bmatrix} B_1^T & D_1^T \\ \ldots & \ldots \\ B_{p+1}^T & D_{p+1}^T \end{bmatrix}.$$

It is known that

$$[A^T] = \begin{bmatrix} F_1^T & V_1^T \\ \ldots & \ldots \\ F_{p+1}^T & V_{p+1}^T \end{bmatrix} = \begin{bmatrix} F_1^T & & \\ & \ddots & \\ & & F_{p+1}^T \end{bmatrix} \begin{bmatrix} V_1^T \\ \vdots \\ V_{p+1}^T \end{bmatrix}.$$

Hence,

$$[A] = \lfloor D_1 \vdots D_2 \vdots \ldots \vdots D_{p+1} \rfloor \begin{bmatrix} B_1 & & \\ & \ddots & \\ & & B_{p=1} \end{bmatrix}$$

$$= [W] \times [Q].$$

In matrix $[W]$, we group the neighboring blocks in pairs:

$$[W] = \lfloor D_1 \cup D_2 \vdots D_3 \cup D_4 \vdots \ldots \vdots D_{p+1} \rfloor$$

$$= \left[d_1 \vdots d_2 \vdots \ldots \vdots d_F \right],$$

where $F = \left\lfloor \frac{N}{2k} \right\rfloor$.

Apparently, matrix $[W] = \left[d_1 \vdots d_2 \vdots \ldots \vdots d_F \right]$ can also be represented as a product of two multipliers. One of them, $[Q_1]$, exhibits block–diagonal structure:

$$[W] = [W_1] \times [Q_1].$$

Matrix W_1 can be transformed in a similar way. Finally, the original matrix can be represented as a product of $h = \lfloor \log_2 N \rfloor + 1$ sparse multipliers:

$$[A] = [D_h]\prod_{i=1}^{h}[C_i], \text{ where } [C_i] = \begin{bmatrix} B_i^{(1)} & & \\ & B_i^{(2)} & \\ & & B_{p=1}^{(p)} \end{bmatrix},$$

$[B_i]$ is a modified matrix corresponding to the ith step of factorization of the given matrix (submatrix) p, and $[D_i]$ is the permutation matrix formed by condition (3) for the ith step of factorization.

4. ALGORITHM OF FACTORIZING RASTER BINARY MATRICES

Using the above definitions and theorems, we can propose an algorithm of factorizing an arbitrary raster binary matrix $[A]$ of size $M \times N$.

(i) Matrix $[A]$ is partitioned into blocks A_j containing pairs of the neighboring columns. The number of blocks is $P = \lfloor (N + 1)/2 \rfloor$.

(ii) For each block A_j, we form an auxiliary matrix $[B_j]$ by eliminating repeating and inverse rows according to Statement 1. The first multiplier of factorization is formed from $[B_j]$ matrices and exhibits block–diagonal structure in agreement with Statement 2:

$$[W] = \mathrm{diag}[B_j].$$

(iii) The second multiplier in factorizing $[D]$ with blocks D_j is derived from auxiliary matrices $[B_j]$ and the blocks of matrix A_j. For this purpose, we compare each row of block A_j with the rows of B_j according to Statement 1 and obtain element (d_{ij}) by expression (4).

(iv) The neighboring blocks of D_j are grouped in pairs, which yields a new partition of matrix $[D]$. The number of nonzero elements in each block does not exceed 2.

(v) Steps 2–4 of the algorithm are repeated. The procedure is terminated after $\lfloor \log_2 N \rfloor$ steps of factorization.

Using the algorithm proposed, one can represent the matrix $[A]$ as a product of sparse matrices–multipliers:

$$A = \begin{bmatrix} 1&1&1&1&1&1 \\ 1&1&1&1&1&1 \\ 1&1&0&0&1&1 \\ 1&1&0&0&1&1 \\ 1&1&1&1&1&1 \\ 1&1&1&1&1&1 \end{bmatrix} = \begin{bmatrix} 1&& \\ &1& \\ &&1 \\ &1& \\ &&1 \\ &&1 \end{bmatrix}\begin{bmatrix} 1&&&1 \\ &1&&1 \\ &&1&0 \end{bmatrix}$$

$$\times\begin{bmatrix} 1&1&&& \\ &1&1&& \\ 0&1&&& \\ &&1&& \\ &&&1& \end{bmatrix}\begin{bmatrix} 1&1&&&& \\ 0&1&&&& \\ &&1&1&& \\ &&&&1&1 \\ &&&&1&0 \end{bmatrix}.$$

In the representation of an object as sparse raster binary matrices in a correlative search for objects in an image, only the first iteration employs the rule of multiplication (3). The output data of the first iteration represent correlation coefficients K for the matrix fragments. The operations of the subsequent iterations employ the rule of multiplication (3):

$$0 \times K = -K, \quad 1 \times K = K. \tag{6}$$

The absence of an information symbol in a position of a sparse binary matrix $[A]$ makes it possible to omit the multiplication by the element (whose position is determined by the VMP regulations) of an intermediate correlation matrix $[H]$. The corresponding position is not taken into account in calculations of the correlation values.

5. ESTIMATION OF THE UPPER BOUND FOR THE COMPUTATIONAL COMPLEXITY OF THE CORRELATIVE SEARCH USING FACTORIZATION OF RASTER MATRICES OF TEMPLATES

Factorization enables one to represent a raster binary matrix of size $N \times N$ as a product of sparse matrices with block–diagonal structure. Here, each of the matrices–blocks contains no more than two information symbols per row. Therefore, the number of additions and subtractions does not exceed the total number of rows in the matrices–multipliers.

In the general case, the maximum number of rows Z in a block of a matrix–multiplier depends on the number of various binary numbers in the block excluding inversions. Note that the number of rows cannot be greater than N. Hence, the number of operations for the ith multiplier is represented as

$$Z_i = \begin{cases} 2^{2^i - 1} \text{ if } 2^{2^i - 1} < N \\ N \text{ if } 2^{2^i - 1} \ge N; \; i = \overline{1, h}; \; h = \lfloor \log_2 N \rfloor. \end{cases}$$

The number of blocks in a matrix is given by

$$P_i = \begin{cases} \left\lceil \dfrac{N}{2^i} \right\rceil \text{ if } N \equiv 0 \bmod 2^i \\ \left\lfloor \dfrac{N}{2^i} \right\rfloor + 1 \text{ if } N \not\equiv 0 \bmod 2^i. \end{cases}$$

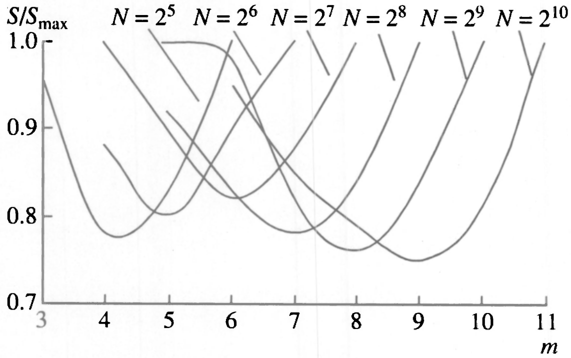


Fig. 1. Plots of the ratio of the current and maximum complexities of VMP calculation versus the number of columns in submatrices for $N \times N$ matrices ($N = 2^n$).

The number of operations necessary for calculating the product of a segment of an input vector–signal by blocks of size $N \times m$ is $\left(\frac{N}{m}\right)k$, where k is the number of additions and subtractions (with allowance for partial sums and inversions) necessary for calculating a product of the segment of the input vector by the block–matrix.

The further stage of multiplying a vector by a matrix by the factorization algorithm implies summation of the products of the input vector segments by the blocks–matrices. Thus, the VMP calculation involves both inner (multiplication of the input vector by the blocks–matrices) and outer (summation of the results of inner calculations) procedures.

The summation of the results of multiplying the input vector segments by the blocks–matrices needs no more than $N\left(\left\lfloor \frac{N}{m} \right\rfloor - 1\right)$ additions and subtractions, where $\lfloor * \rfloor$ is the nearest integer, N is the number of rows (columns) in the original square matrix, and m is the number of columns in the block–matrix. With allowance for expression $\left\lfloor \frac{n}{2} \right\rfloor < m < n$, the total number of additions and subtractions necessary for calculating VMP is represented as [12]

$$C \approx K\frac{N}{m} + \left(N\left\lfloor \frac{N}{m} \right\rfloor - 1\right),$$

where $n = \log_2 N$.

The computational complexity is defined as

$$S = \frac{K}{m} + \left(\left\lfloor \frac{N}{m} \right\rfloor - 1\right). \quad (7)$$

It is seen from this expression that the computational complexity substantially depends on m for the given size of the matrix.

To determine an optimum size m of the block for the given size of matrix $N \times N$, we use expression (7) as an objective function. The terms of a series $K = f(m)$ [13] are well approximated by a function $K = 0.91 \times 2^{0.93m}$. In this case, the objective function S is written as

$$S(m) = \frac{0.91 \times 2^{0.93m}}{m} + \left(\left\lfloor \frac{N}{m} \right\rfloor - 1\right). \quad (8)$$

The optimization of partitioning into blocks is reduced to determining the minimum of the computational complexity $S = f(m)$ for the given N.

Let us differentiate expression (8):

$$\frac{dS}{dm} = \frac{0.91 \times 2^{0.93m} \times 0.93\ln 2 \times m - 0.91 \times 2^{0.93m}}{m^2}$$

$$- \frac{N}{m^2} = \frac{(0.59m - 0.91)2^{0.93m} - N}{m^2}.$$

The minimum of function $S = f(m)$ is attained for $\frac{dS}{dm} = 0$. Therefore,

$$(0.59m - 0.91)2^{0.93m} = N. \quad (9)$$

By taking the logarithm of expression (9) and approximating $\log_2(0.59m - 0.91)$ by the least-squares method with a linear function $(Am + B)$ for $m \in [3 \div 10]$, we obtain

$$0.93m + 0.377m - 1.099 = \log_2 N.$$

Then, expression

$$m = 0.765\log_2 N + 0.84 \quad (10)$$

determines the optimum size of the block for the given N.

Of practical interest are only the integer values of m nearest to the optimum obtained. Therefore, for determining the real size, expression (10) can be rewritten as

$$m = |0.765\log_2 N + 0.84| = n - 1. \quad (11)$$

A graphical analysis of the dependence of computational complexity of VMP calculations on various quantities for the given size of the square matrix (Fig. 1) proves the validity of expression (11).

Table 1 summarizes the data regarding the number of operations necessary for calculating 2D correlation by various methods based on double transforms and by direct methods of matrix multiplication. Analysis of the data presented shows that the use of factorization in calculations of 2D correlation decreases computational costs (in comparison to the basic methods) for templates of a size no greater than 256 × 256.

Table 1. The number of additions and subtractions in calculation of 2D correlation

$N \times N$	Agarval–Cooley algorithm	Reider–Breiner FFT	Algorithm based on polynomial transforms	Nussbaumer algorithm	Algorithm employing NFPT and OFFFTSB*	Direct approach of matrix multiplication	Algorithm based on factorization	Algorithm based on factorization and partitioning into blocks
16 × 16	11 580	8192	6676	6628	6844	3840	1792	1792
30 × 30	60 260	–	–	–	–	26 100	9450	9450
32 × 32	–	46 336	35 828	35 800	35 516	31 744	10 240	10 240
48 × 48	–	–	–	–	–	108 288	34 560	31 795
60 × 60	384 500	–	–	–	–	212 400	64 800	56 880
64 × 64	–	241 152	196 212	194 820	174 780	258 040	77 824	68 812
72 × 72	700 352	–	–	–	–	368 064	108 864	97 459
80 × 80	1 134 750	–	–	–	–	505 600	147 200	126 720
120 × 120	2 927 040	–	–	–	–	1 713 600	457 200	379 200
128 × 128	–	1 190 912	955 342	948 100	830 140	2 080 768	524 288	463 667
240 × 240	–	–	–	–	–	13 766 400	2 822 400	2 633 142
256 × 256	–	5 675 008	–	4 739 204	3 844 796	16 711 680	3 342 336	3 099 852

* Algorithm employing Nussbaumer fast polynomial transforms and odd-frequency fast Fourier transform with split base.

6. COMPUTATIONAL COSTS OF PROCESSING REAL IMAGES

For estimating the real computational costs, we used images of various structural complexity shown in Fig. 2. Table 2 summarizes experimental data.

Analysis of the results shows that the real computational costs of using matrix factorization is a few times less than the maximum costs necessary for images of arbitrary sizes. The benefits depend on the image structure: the higher the regularity of the structure, the greater the benefit.

The proposed algorithm allows one to detect objects of arbitrary sizes in images of various format. The computational costs of calculating the correlation functions depend on the format of the image processed, the size of the object, and the number of operations necessary for multiplying a vector by a factorized matrix of the object. It is expedient to consider an optimum represen-

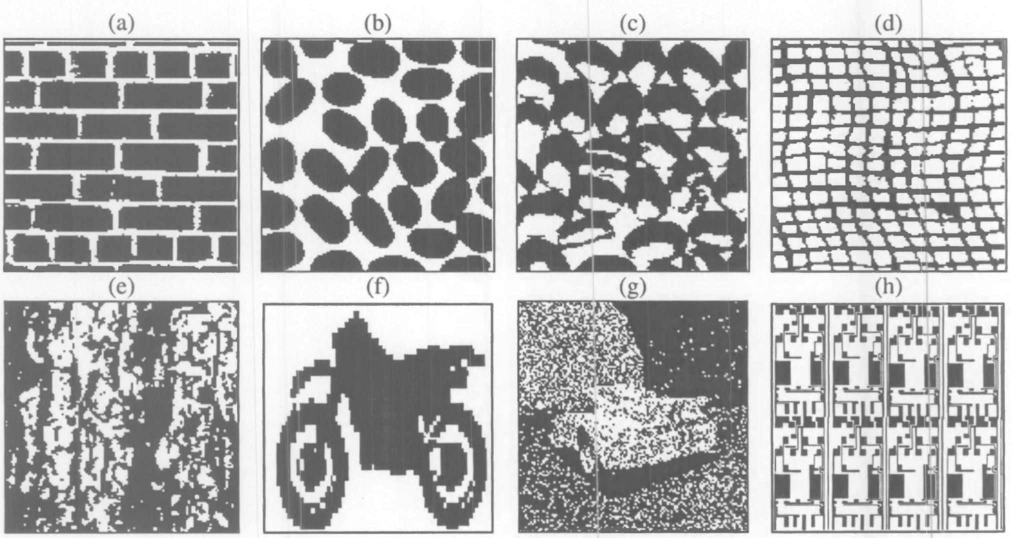


Fig. 2. Images with various structure regularity.

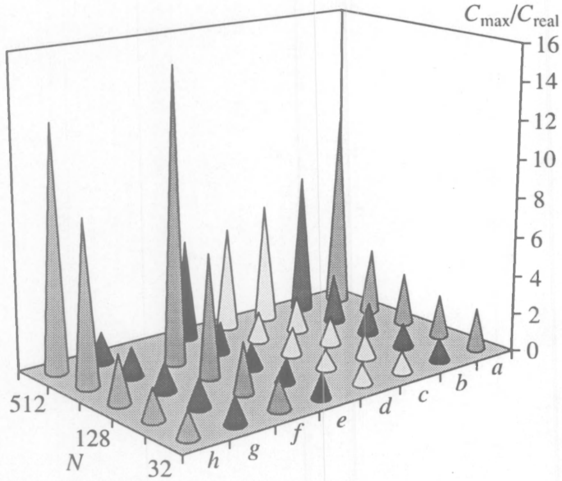


Fig. 3. A decrease in the computational costs relative to the upper bound.

tation of the format and template. Based on the experimental data, we present the number of operations necessary for a correlative search for the objects of book and album formats (see Table 3).

The results obtained show that minimum computational costs correspond to the search for an object of book format in the image. To reduce the computational costs of the search for rectangular objects, one should use album format.

7. EXAMPLES OF THE PRACTICAL APPLICATION OF THE ALGORITHMS

Nowadays, the search for objects by comparison with the template is used in controlling photomasks, chips, and printed boards. It is known that the quality of manufacturing of photomasks is better controlled than that of chips or printed boards. Therefore, the quality control of photomasks can employ the correlation method of comparing the image processed and the template. This method implements all the advantages of the maximum-likelihood method and enables one to detect objects with minimum deviations from the template. A high-quality photomask should be identical to the template. After preprocessing, the photomasks are presented in binary format. Thus, the photomasks can be classified as high-quality or low-quality ones based on the above correlation algorithm of the search for the objects. The computational costs can be decreased by preliminarily factorizing the raster matrices of the tem-

Table 2. Real number of operations in calculation of 2D correlation

Type of the image processed	$N \times N$				
	32 × 32	64 × 64	128 × 128	256 × 256	512 × 512
(a)	4832	30 592	174 464	906 752	2 138 624
(b)	9248	54 400	278 272	1 300 736	2 918 912
(c)	9408	55 744	312 832	1 558 016	3 500 032
(d)	8800	60 288	350 336	1 807 360	4 035 072
(e)	8448	56 256	342 912	1 871 360	4 203 008
(f)	5859	26 752	73 710	205 275	576 919
(g)	8448	55 616	370 688	2 472 960	16 204 800
(h)	7296	42 088	184 832	695 808	1 713 152

Table 3. Computational costs of the search for rectangular objects

Object size	Image size			
	400 × 300	300 × 400	700 × 300	300 × 700
32 × 216	**36 006 000**	58 774 500	**63 010 500**	154 084 500
216 × 32	48 806 000	48 154 500	81 910 500	87 305 500
54 × 216	**46 614 000**	76 090 500	**81 574 500**	199 480 500
216 × 54	81 312 400	85 674 300	142 296 700	159 744 300
108 × 216	**64 512 000**	105 984 000	**112 896 000**	278 764 000
216 × 108	122 593 600	139 585 200	214 538 800	282 505 200

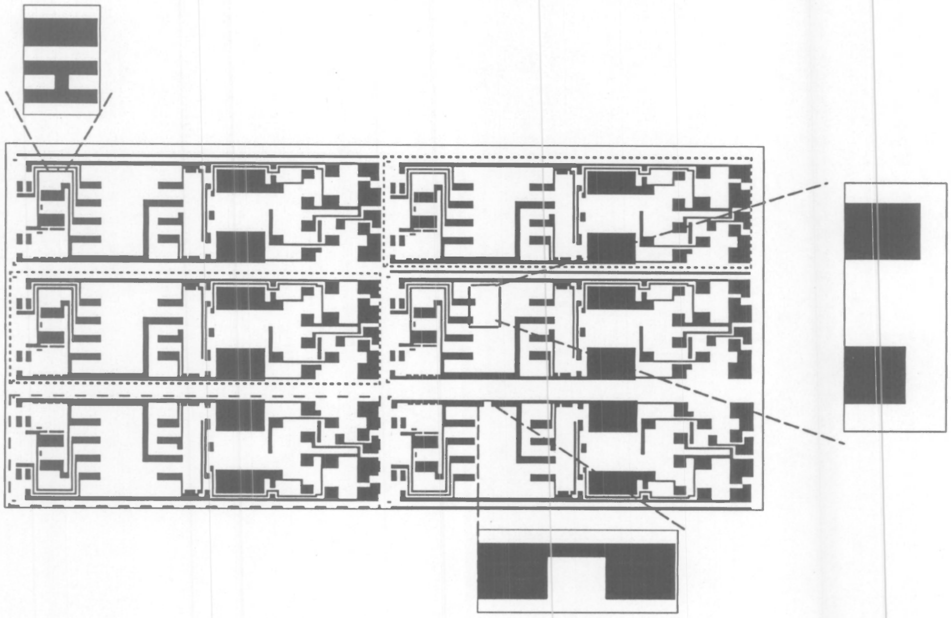


Fig. 4. Results of the search for high-quality photomasks.

plates. The factorization makes it possible to store raster matrices of binary images as sparse matrices–multipliers, which significantly reduces the computational costs.

Figure 4 shows the results of the search for high-quality photomasks based on calculating the correlation function of the fragments of the image processed and the template photomask (Fig. 5). The correlation coefficient is assumed to be equal to unity. In this case, one can detect only objects completely identical to the template. Figure 4 also shows enlarged low-quality fragments of photomasks.

In practice, it is sometimes necessary to detect an object in the presence of noise or interference. Generally, the search for objects in a noisy image can be defined as comparison of a certain threshold number to another number obtained by mathematical transformation of the description of the analyzed fragment and the template. For the proposed algorithm employing factorization of matrices, the decision regarding the presence of the object is made from expressions (1) and (2). The adequacy of object detection is characterized by the probabilities of nondetection and false detection. False detection owing to a blurred correlation peak is typical of gray-scale images. Its probability can be diminished by normalizing mutual correlation of the template and image fragment. For binary images, the maximum of the correlation function is much sharper. In addition, the probability of false detection substantially depends on the presence of extraneous objects similar to the template. The probability of nondetection depends on the noise intensity and the threshold value. Filtration algorithms can diminish the noise intensity, although the corresponding procedures increase the computational time.

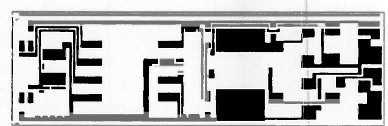
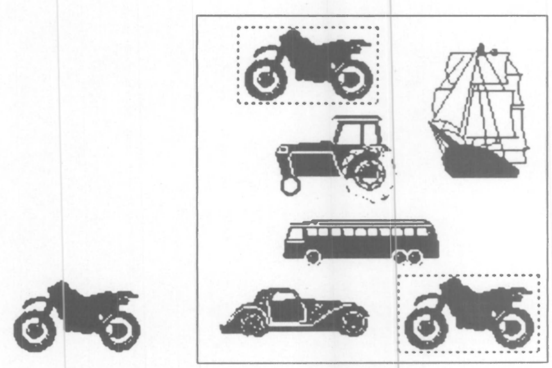


Fig. 5. Template photomask.



(a) (b)

Fig. 6. Results of detecting arbitrary objects: (a) object and (b) image processed.

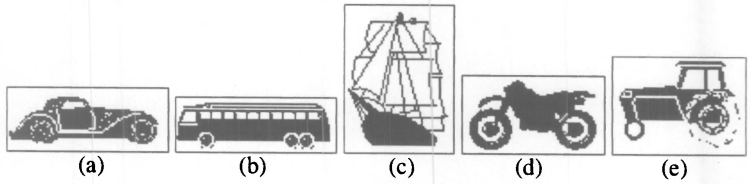


Fig. 7. Templates of objects for detection.

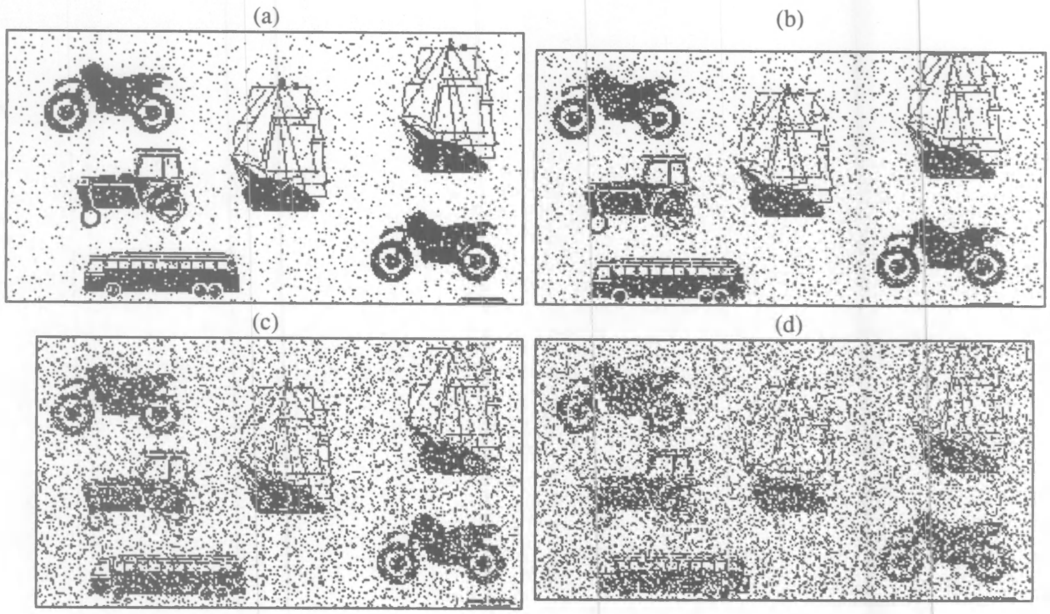


Fig. 8. Noisy images with the noise amplitude (a) 5, (b) 10, (c) 20, and (d) 30%.

That is why it is expedient to study the possibility of correct detection of binary objects in the image by factorizing matrices. Figure 7 shows template objects used for experiments on the search for objects in images with normal distribution of noise.

Tables 4–7 demonstrate the results of experiments on detecting objects in the presence of normal noise for various thresholds and noise intensities.

The data presented show that for the normal distribution of noise with the amplitude less than 10%,

Table 4. The probability of object detection for images with 5% of noise

Template position \ Threshold level	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.7	0.75	0.8
A	–	0.3	0.38	0.71	1	1	1	1	1	0.42
B	–	0.53	0.85	1	1	1	1	1	1	0.44
C	0.54	1	1	1	1	1	1	0.91	0.5	0.08
D	–	–	–	0.25	0.44	0.92	1	1	0.88	0.25
E	–	–	–	0.43	0.9	1	1	1	0.9	0.43

Table 5. The probability of object detection for images with 10% of noise

Template position \ Threshold level	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
A	–	–	–	0.76	0.98	1	1	0.92	0.85	0.22
B	–	–	0.77	1	1	1	1	1	0.85	–
C	0.7	1	1	1	1	0.91	0.83	–	–	–
D	–	–	–	0.58	1	1	0.58	–	–	–
E	–	–	0.69	0.88	1	1	0.93	–	–	–

Table 6. The probability of object detection for images with 20% of noise

Template position \ Threshold level	0.2	0.22	0.25	0.3	0.33	0.35	0.36	0.37	0.38	0.4
A	–	–	–	0.72	0.91	1	1	0.98	0.97	0.94
B				0.95	0.97	1	1	1	1	0.97
C	0.92	1	1	1	1	0.91	0.75	–	–	–
D	–	–	–	–	–	0.74	0.98	1	1	0.85
E	–	–	–	0.75	0.88	1	1	1	1	0.93

Table 7. The probability of object detection for images with 30% of noise

Template position \ Threshold level	0.1	0.13	0.15	0.16	0.17	0.18	0.23	0.25	0.26	0.27	0.28	0.29	0.3
A	–	–	–	–	–	–	–	–	0.93	0.94	0.96	0.92	0.88
B	–	–	–	–	–	–	–	0.85	0.93	0.97	0.97	0.91	0.82
C	0.71	0.94	0.98	0.98	0.92	0.92	–	–	–	–	–	–	–
D	–	–	–	–	–	–	–	0.65	0.82	0.94	0.94	0.92	0.85
E	–	–	–	–	–	–	0.88	0.93	0.88	0.84	–	–	–

correct detection is possible for a certain interval of the threshold values.

9. CONCLUSION

Substantial computational complexity and time costs are typical of correlative processing of images. The computational costs are reduced by using fast processing algorithms based on factorization of the original matrices.

In this work, we present factorization of binary image matrices of arbitrary sizes and structures for (1, 0)-alphabet representation of the raster data. An algorithm for the factorization of raster matrices of binary images is proposed. It is based on the sequential elimination of repeating and inverse (in terms of representation of the raster binary data) rows.

We obtained an expression for estimating the upper bound of the computational complexity of VMP by matrix factorization, optimized block partitioning of matrices for factorization, and determined real computational costs of the correlative search for objects in images.

It is obvious from experimental data that the real computational costs of calculating 2D correlation during image processing by matrix factorization are a few times less than the maximum ones necessary for processing images of arbitrary sizes. The benefits depend on the image structure and increase when the structure regularity increases.

We developed a correlation algorithm of the search for binary objects that is invariant with respect to rotation by 180° relative to the horizontal axis.

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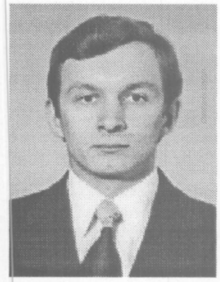
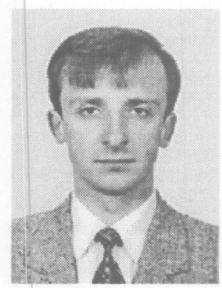
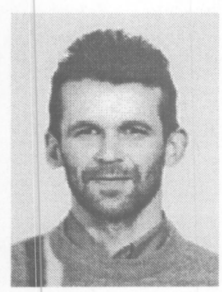


Sergey Ablameyko. Graduated from Belarus State University in 1978. Received his PhD (Kandidat Nauk) Degree in 1984 and Doctoral Degree in 1990. Received the title of Professor in 1992. Deputy Director of the Institute of Engineering Cybernetics, Belarus Academy of Sciences, half-time Professor at the University of Informatics and Radioelectronics. Author of approximately 160 works, including six books on image processing. Advisory Editor of International Journal of Machine Graphics and Vision. Scientific interests: document image analysis, image processing, object representation and recognition, digital geometry, knowledge-based systems, geographic information systems. Fellow of IEE, Senior Member of IEEE, Fellow of IAPR, Fellow of two Academies, Chairmen of the IEE Belarus Center, Member of the Governing Board of the International Association for Pattern Recognition, President of the Belarus Association for Image Analysis and Recognition, Vice-President of the Belarus Association for Artificial Intelligence.



Rikhard P. Bogush. Born 1974. Graduated from Polotsk State University in 1997. Senior Lecturer of the Department of Radioelectronics, Polotsk State University. Area of research: correlation processing of images and signals. Author of 20 articles.



Sergei V. Mal'tsev. Born 1961. Graduated from Leningrad Institute of Aviation Instrument Making in 1984. Received his PhD (Kandidat Nauk) Degree in 1995. Head of the Department of Radioelectronics, Polotsk State University. Area of research: processing of binary signals and images, nonlinear series for communication systems, and industrial electronics. Author of 26 articles.