ON VECTOR FIELD DEFINED BY THE HOPF MAP $S^3$ ON $S^2$

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The subject of our consideration is the system of ODE’s in $E^3$-space

$$\frac{dx(t)}{dt} = 8 \frac{4zx - y(x^2 + y^2 + z^2) + 4y}{(x^2 + y^2 + z^2 + 4)^2}, \quad \frac{dy(t)}{dt} = 8 \frac{4zy + x(x^2 + y^2 + z^2) - 4x}{(x^2 + y^2 + z^2 + 4)^2},$$

$$\frac{dz(t)}{dt} = \frac{24x^2 + 24y^2 - 8 \bar{z}^2 - (x^2 + y^2 + z^2)^2}{(x^2 + y^2 + z^2 + 4)^2},$$

associated with the Hopf map $S^3 \to S^2$ of three dimensional sphere with equation $z_1\bar{z}_1 + z_2\bar{z}_2 = 1$ on two-dimensional sphere $S^2$ considered as set of the points $[z_1/z_2, 1]$ of complex space $C^2$.

In our report we discuss the properties of this system of equations. The results presented generalize those obtained in [1].

**Theorem.** In the spherical system of coordinates $x(t) = r(t)\cos(\phi(t))\sin(\psi(t)), \quad z(t) = r(t)\cos(\psi(t)), \quad y(t) = r(t)\sin(\phi(t))\sin(\psi(t))$ the system takes form

$$\frac{dr(t)}{dt} = \frac{(r(t))^4 + 8 (r(t))^2 - 64 (r(t))^2\sin(\psi(t))^2 + 16)\cos(\psi(t))}{(r(t))^4 + 8 (r(t))^2 + 16},$$

$$\frac{d\phi(t)}{dt} = -\frac{4 + (r(t))^2}{(r(t))^4 + 8 (r(t))^2 + 16},$$

$$\frac{d\psi(t)}{dt} = \frac{\sin(\psi(t))(r(t))^4 + 40 (r(t))^2 - 64 (r(t))^2\sin(\psi(t))^2 + 16)}{(r(t))^4 + 8 (r(t))^2 + 16},$$

and its solutions are expressed through the function $\sqrt{H(r)}$ which satisfies to the equation

$$4C_1 \text{Bessel} I(0, 1/2 \sqrt{H(r)r^2}) + C_1 \text{Bessel} I(0, 1/2 \sqrt{H(r)r^2})r^2 -$$

$$-8C_1 \text{Bessel} I(1, 1/2 \sqrt{H(r)r^2})\sqrt{H(r)r^2} + 4 \text{Bessel} K(0, -1/2 \sqrt{H(r)r^2}) +$$

$$+ \text{Bessel} K(0, -1/2 \sqrt{H(r)r^2})r^2 - 8 \text{Bessel} K(1, -1/2 \sqrt{H(r)r^2})\sqrt{H(r)r^2} = 0,$$

where $\sqrt{H(r)} = \sin(\psi(r))$ and $C_1$ are the parameters.

More detail information about properties of the solutions can be obtained by means of the first order p. d. e. associated with the considered systems of equations [2].

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**References**