ON VECTOR FIELD DEFINED BY THE HOPF MAP S^3 ON S^2

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The subject of our consideration is the system of ODE's in E^3-space

$$\frac{d}{dt}x(t) = 8\,\frac{4\,zx - y(x^2+y^2+z^2)+4\,y}{(x^2+y^2+z^2+4)^2}, \quad \frac{d}{dt}y(t) = 8\,\frac{4\,zy + x(x^2+y^2+z^2)-4\,x}{(x^2+y^2+z^2+4)^2},$$

$$\frac{d}{dt}z(t) = \frac{24\,x^2+24\,y^2-8\,z^2-(x^2+y^2+z^2)^2-16}{(x^2+y^2+z^2+4)^2},$$

associated with the Hopf map $S^3 \to S^2$ of three dimensional sphere with equation $z_1\bar{z}_1 + z_2\bar{z}_2 = 1$ on two-dimensional sphere S^2 considered as set of the points $[z_1/z_2, 1]$ of complex space C^2.

In our report we discuss the properties of this system of equations. The results presented generalize those obtained in [1].

Theorem. *In the spherical system of coordinates* $x(t) = r(t)\cos(\phi(t))\sin(\psi(t))$, $z(t) = r(t)\cos(\psi(t))$, $y(t) = r(t)\sin(\phi(t))\sin(\psi(t))$ *the system takes form*

$$\frac{d}{dt}r(t) = \frac{((r(t))^4 + 8\,(r(t))^2 - 64\,(r(t))^2(\sin(\psi(t)))^2 + 16)\cos(\psi(t))}{(r(t))^4 + 8\,(r(t))^2 + 16},$$

$$\frac{d}{dt}\phi(t) = -8\,\frac{-4+(r(t))^2}{(r(t))^4 + 8\,(r(t))^2 + 16},$$

$$\frac{d}{dt}\psi(t) = -\frac{\sin(\psi(t))((r(t))^4 + 40\,(r(t))^2 - 64\,(r(t))^2(\sin(\psi(t)))^2 + 16)}{r(t)((r(t))^4 + 8\,(r(t))^2 + 16)}$$

and it solutions are expressed trough the function $\sqrt{H(r)}$ *which satisfies to the equation*

$$4\,C_1\,\mathrm{Bessel}\,I(0,1/2\,\sqrt{H(r)r^2}) + C_1\,\mathrm{Bessel}\,I(0,1/2\,\sqrt{H(r)r^2})r^2 -$$

$$-8\,C_1\,\mathrm{Bessel}\,I(1,1/2\,\sqrt{H(r)r^2})\sqrt{H(r)r^2} + 4\,\mathrm{Bessel}\,K(0,-1/2\,\sqrt{H(r)r^2}) +$$

$$+\mathrm{Bessel}\,K(0,-1/2\,\sqrt{H(r)r^2})r^2 - 8\,\mathrm{Bessel}\,K(1,-1/2\,\sqrt{H(r)r^2})\sqrt{H(r)r^2} = 0,$$

where $\sqrt{H(r)} = \sin(\psi(r))$ *and* C_i *are the parameters.*

More detail information about properties of the solutions can be obtained by means of the first order p. d. e. associated with the considered systems of equations [2].

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References

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2. Dryuma V. S. *On the theory of the first order systems of differential equations* // Intern. Conf. on Differential Equations and Dynamical Systems. Suzdal, July' 2–7, 2010.Abstracts. Suzdal, 2010. P. 205–206.