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INVESTIGATION OF THE MOISTURE INFLUENCE ON SOIL MASS STABILITY ON THE SLOPE

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To determine the effect of moisture on the stability of the soil mass, we divide the problem into two parts: 1) determination the saturation change in soil due to pipeline damage, and 2) optimization problem of searching the minimum stability factor and the critical slip surface.

First we considered two-dimensional problem of soil slope saturation (domain Ω) as a result of the pipeline damage (point D, $D \in \partial\Omega$, where $\partial\Omega$ is a border of Ω). Therefore it is obtained two subdomains of complete and incomplete saturated soil. Our task is to determine the dynamics of saturation region with time.

For this purpose Richards' equation was used [1] with regard to saturation function $s(x,z,t) = (\theta - \theta_{\min})/(\theta_{\max} - \theta_{\min})$, where $\theta_{\min}$ is the residual liquid content in the porous medium; $\theta_{\max}$ is the saturated liquid content in a porous medium; $0 \leqslant \theta_{\min} \leqslant \theta \leqslant \theta_{\max} = \sigma$, σ is a porosity and θ is the volumetric liquid content in the unit volume of a soil.

Then the mathematical model of the investigated process can be written by the following boundary-value problem:

$$\frac{\partial s(x,z,t)}{\partial t} = \frac{\partial}{\partial x}\left(D(s)\frac{\partial s(x,z,t)}{\partial x}\right) + \frac{\partial}{\partial z}\left(D(s)\frac{\partial s(x,z,t)}{\partial z}\right) - \frac{1}{\theta_{\max} - \theta_{\min}} \cdot \frac{\partial K(s)}{\partial z},$$

$$(x,z) \in \Omega, \quad t \in (0;T];$$

$$s(x,z,0) = s_0(x,z), \quad (x,z) \in \overline{\Omega} = \Omega \cup \Gamma;$$

$$s(x,z,t)|_{\Gamma_1} = s_1(x,z,t), \quad (x,z) \in \Gamma_1; \quad \left(-D(s)\frac{\partial s(x,z,t)}{\partial z}\right)\bigg|_{\Gamma_2} = 0, \quad (x,z) \in \Gamma_2, \quad t > 0;$$

$$\frac{\partial s(x,z,t)}{\partial x}\bigg|_{\Gamma_3 \setminus \{D\}} = 0, \quad (x,z) \in \Gamma_3 \setminus \{D\}; \quad s(x,z,t)|_D = 1, \quad (x,z) \in D, \quad t > 0,$$

where $D(s)$ is the soil liquid diffusivity that can be treated as a nonlinear diffusion coefficient; $K(s)$ is a hydraulic conductivity of an unsaturated porous medium; $s_1(x,z,t)$

is a given function. Functions $D(s)$ and $K(s)$ were determined according to the following two models: 1) Brooks-and-Corey model; 2) Mualem-Van Genuchten model [1].

The numerical solution of above boundary-value problem was founded by meshfree radial basis function method [2].

Using the solutions of the above problem at the second part we found the minimum slope stability factor and possible slip surface using engineering methods such as Mozhevitinova, Fedorovsky — Kurylo, circular cylindrical sliding surfaces method, etc. [3], and optimization techniques for minimizing the objective function of many variables, such as coordinate descent method and golden section search.

As a result of these two steps it can be predicted what amount of moisture in the slope soil will lead to stability loss and dangerous geological processes such as landslides, avalanches, mudflows and so on.

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ON SOME APPLICATIONS FOR EQUATIONS OF ELLIPTIC TYPE WITH SPECTRAL PARAMETER AND DISCONTINUOUS NONLINEARITY

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Models with discontinuous nonlinearities are used as idealization of continuous processes such that nonlinear parameters rapidly vary on small intervals in the range of phase variables. For many problems of hydrodynamics, thermophysics, electrophysics, control theory, mathematical biology, and other fields of modern natural science the mathematical models often include differential equations with discontinuous nonlinearities. As an examples, we can mention the Gol'dshtik's and the Lavrent'ev's problems for separated flows of incompressible fluid [1, 2]. Also, we can remember the Kuiper's problem on heating of conductor when voltage and temperature are constant on its surface, and electroconductivity of material changes jumpwise as a function of the temperature [3].

We consider the Gol'dshtik model for separated flows in incompressible fluid. Besides, a model problem approximately describing the flow of viscous incompressible fluid in a square cavern is solved. A solution of this two-dimensional problem of mathematical physics for a finite domain is found numerically using the Partial Differential Equation Toolbox of the MATLAB system by the finite element method. Estimations of differential operator for the problem are calculated. The existence of semiregular solutions of the Gol'dshtik problem is proved. Such solutions are of great interest in applied problems. The result on the number of solutions to the Gol'dshtik problem is gained by a variational method.

We study the Lavrent'ev mathematical model for separated flows with an external perturbation. This model consists of a differential equation with discontinuous nonlinearity