ANALYTICAL MODEL OF THE PERIODONTAL LIGAMENT BASED ON NONLINEAR THEORY OF THE SQUEEZE SHELLS

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Teeth are surrounded by the periodontal ligament, which is a thin membrane. It consists of the collagen fibers that provide attachment of the tooth to the alveolar bone. The contact between the tooth root and the alveolar bone is absent under normal conditions. Action of load on the tooth crown is transmitted to the alveolar bone by the strains of the periodontal ligament.

In most cases the analytical modeling of the periodontal membrane is performed with the assumption that the geometric shape of the tooth root is described by the elliptical (circular) paraboloid or hyperboloid [1, 2]. Such an approximation of the real shape of the tooth root gives sufficient accuracy in calculating the single-root teeth movements [3]. The internal surface of the shell periodontal coincides with the outer surface of the tooth root. It is assumed that the root of the tooth is a solid. The outer surface of the periodontal ligament is obtained by displacement its inner surface normal to the surface of the tooth root or in the vertical direction. The displacement of the inner surface points of the periodontium coincides with the displacement of the tooth root. The translational displacement and rotation angles of the tooth root are determined on the base of the root equilibrium equations with taking into account the different simplifying assumptions. A mathematical model for calculating the stress-strain state of the periodontal membrane without idealizing assumptions isn't developed. The purpose of this study is to develop a mathematical model of the periodontal ligament in the form of a circular paraboloid based on the nonlinear theory of thin shells with taking into account its squeezing [4].

The resulting system contains nine differential equations in partial derivatives (three equations of equilibrium and six equations of state of the shell). Unknowns are the components of the stress tensor and the components of the displacement vector of the shell. The boundary conditions correspond to the translational motion of the tooth root in the horizontal plane (the plane is perpendicular to the axis of the paraboloid). Displacements of the shell internal surface coincide with the displacements of the tooth root (the root of the tooth is modeled by solid). Displacements of the external surface of the shell are equal to zero (the shell is rigidly fixed in the alveolar bone). The periodontal tissue is modeled by the isotropic linear elastic material. The solution of the system of equations for the periodontal shell with the above boundary conditions is carried out based on the base of the asymptotic method.

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