One-Dimensional and Multi-Dimensional Integral Transforms of Buschman–Erdélyi Type with Legendre Functions in Kernels

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Abstract

This paper consists of two parts. In the first part we give a brief survey of results on Buschman–Erdélyi operators, which are transmutations for the Bessel singular operator. Main properties and applications of Buschman–Erdélyi operators are outlined. In the second part of the paper we consider multi-dimensional integral transforms of Buschman–Erdélyi type with Legendre functions in kernels. Complete proofs are given in this part, main tools are based on Mellin transform properties and usage of Fox H-functions.

Keywords

Buschman–Erdélyi operators Multidimensional Buschman–Erdélyi operators Transmutations Mellin transform Fox H-function

References

1. 1.

   H. Bateman, A. Erdélyi, *Higher Transcendental Functions*, vol. 1 (McGraw-Hill, New York, 1953)zbMATHGoogle Scholar

2. 2.

   H. Begehr, R.P. Gilbert, *Transformations, Transmutations and Kernel Functions*, vols. 1, 2, Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 59 (Longman Scientific & Technical, Harlow, 1993)Google Scholar

3. 3.

   Yu. A. Brychkov, H.-Y. Glaeske, A.P. Prudnikov, V.K. Tuan, *Multidimensional Integral Transformations* (Gordon and Breach, Philadelphia, 1992)zbMATHGoogle Scholar

4. 4.

R.G. Buschman, An inversion integral for a Legendre transformation. Amer. Math. Mon. **69**(4), 288–289 (1962)CrossRefGoogle Scholar

5. 5.

R.G. Buschman, An inversion integral for a general Legendre transformation. SIAM Rev. **5**(3), 232–233 (1963)MathSciNetCrossRefGoogle Scholar

6. 6.

R.W. Carroll, *Transmutation and Operator Differential Equations*. Mathematics Studies, vol. 37 (North Holland, Amsterdam, 1979)Google Scholar

7. 7.

R.W. Carroll, *Transmutation, Scattering Theory and Special Functions*. Mathematics Studies, vol. 69 (North Holland, Amsterdam, 1982)Google Scholar

8. 8.

R.W. Carroll, *Transmutation Theory and Applications*. Mathematics Studies, vol. 117 (North Holland, Amsterdam, 1985)Google Scholar

9. 9.

R.W. Carroll, R.E. Showalter, *Singular and Degenerate Cauchy Problems* (Academic, New York, 1976)Google Scholar

10. 10.

I. Dimovski, Operational calculus for a class of differential operators. C. R. Acad. Bulg. Sci. **19**(12), 1111–1114 (1966)MathSciNetGoogle Scholar

11. 11.

I. Dimovski, On an operational calculus for a differential operator. C. R. Acad. Bulg. Sci. **21**(6), 513–516 (1968)MathSciNetzbMATHGoogle Scholar

12. 12.

I. Dimovski, *Convolutional Calculus* (Kluwer, Dordrecht, 1990)CrossRefGoogle Scholar

13. 13.

I.H. Dimovski, V.S. Kiryakova, Transmutations, convolutions and fractional powers of Bessel-type operators via Meijer's *G*-function, in *Complex Analysis and Applications '83. Proceedings of International Conference Varna 1983*, Sofia (1985), pp. 45–66Google Scholar

14. 14.

A. Erdélyi, An integral equation involving Legendre functions. SIAM Rev. **12**(1), 15–30 (1964)MathSciNetzbMATHGoogle Scholar

15. 15.

A. Erdélyi, Some integral equations involving finite parts of divergent integrals. Glasgow Math. J. **8**(1), 50–54 (1967)MathSciNetCrossRefGoogle Scholar

16. 16.

A. Erdelyi, W. Magnus, F. Oberhettinger, F.G. Tricomi, *Higher Transcendental Functions*, vol. 1 (McGraw-Hill, New York, 1953)zbMATHGoogle Scholar

17. 17.

D.K. Fage, N.I. Nagnibida, *The Problem of Equivalence of Ordinary Differential Operators* (Nauka, Novosibirsk, 1977, in Russian)Google Scholar

18. 18.

S. Helgason, *Groups and Geometric Analysis: Radon Transforms, Invariant Differential Operators and Spherical Functions* (Academic, Cambridge, 1984)zbMATHGoogle Scholar

19. 19.

V.V. Katrakhov, S.M. Sitnik, A boundary-value problem for the steady-state Schrödinger equation with a singular potential. Soviet Math. Dokl. **30**(2), 468–470 (1984)zbMATHGoogle Scholar

20. 20.

V.V. Katrakhov, S.M. Sitnik, Factorization method in transmutation theory, in *Non-classical and Mixed Type Equations* (in memory of B.A. Bubnov, Ed. V.N. Vragov) (1990, in Russian), pp. 104–122Google Scholar

21. 21.

V.V. Katrakhov, S.M. Sitnik, Composition method for constructing *B*-elliptic, *B*-hyperbolic, and *B*-parabolic transformation operators. Russ. Acad. Sci., Dokl. Math. **50**(1), 70–77 (1995)Google Scholar

22. 22.

V.V. Katrakhov, S.M. Sitnik, Estimates of the Jost solution to a one-dimensional Schrodinger equation with a singular potential. Dokl. Math. **51**(1), 14–16 (1995)MathSciNetzbMATHGoogle Scholar

23. 23.

V.V. Katrakhov, S.M. Sitnik, The transmutation method and boundary-value problems for singular elliptic equations. Contemp. Math. Fund. Direct. **64**(2), 211–426 (2018, in Russian)Google Scholar

24. 24.

A.P. Khromov, Finite-dimensional perturbations of Volterra operators. J. Mater. Sci. **138**(5), 5893–6066 (2006)MathSciNetGoogle Scholar

25. 25.

A.A. Kilbas, M. Saigo, *H-Transforms. Theory and Applications* (Chapman and Hall, Boca Raton, 2004), 400 pp.Google Scholar

26. 26.

A.A. Kilbas, O.V. Skoromnik, Integral transforms with the Legendre function of the first kind in the kernels on $L_{v,r}$-spaces. Integral Transform. Spec. Funct. **20**(9), 653–672 (2009)MathSciNetCrossRefGoogle Scholar

27. 27.

A.A. Kilbas, O.V. Skoromnik, Solution of a multidimensional integral equation of the first kind with the Legendre function in the kernel over a pyramidal domain. Dokl. Math. **80**(3), 847–851 (2009)MathSciNetCrossRefGoogle Scholar

28. 28.

A.A. Kilbas, O.V. Skoromnik, Integral transforms with the legendre function of the first kind in the kernels on $Lv-,r-L\bar{v},\bar{r}$ -spaces. Integral Transform. Spec. Funct. **20**(9), 653–672 (2009)Google Scholar

29. 29.

A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Applications of Fractional Differential Equations* (Elsevier, Amsterdam, 2006), 523 pp.zbMATHGoogle Scholar

30. 30.

I.A. Kipriyanov, *Singular Elliptic Boundary-Value Problems* (Nauka–Physmatlit, Moskow, 1997, in Russian)Google Scholar

31. 31.

V. Kiryakova, *Generalized Fractional Calculus and Applications*. Pitman Research Notes in Mathematics, vol. 301 (Longman Scientific & Technical, Harlow, Co-publisher Wiley, New York 1994)Google Scholar

32. 32.

V.V. Kravchenko, *Pseudoanalytic Function Theory* (Birkhäuser Verlag, Basel, 2009)CrossRefGoogle Scholar

33. 33.

A. Kufner, L.-E. Persson, *Weighted Inequalities of Hardy Type* (World Scientific, River Edge, 2003)CrossRefGoogle Scholar

34. 34.

B.M. Levitan, *Generalized Translation Operators and Some of Their Applications* (Israel Program for Scientific Translations, Jerusalem, 1964)Google Scholar

35. 35.

B.M. Levitan, *Generalized Translation Operators Theory* (Nauka, Moscow, 1973, in Russian)Google Scholar

36. 36.

B.M. Levitan, *Inverse Sturm−Liouville Problems* (VNU Science Press, Utrecht, 1987)CrossRefGoogle Scholar

37. 37.

J.L. Lions, *Equations differentielles operationnelles et problémes aux limites* (Springer, Berlin, 1961)CrossRefGoogle Scholar

38. 38.

V.A. Marchenko, *Spectral Theory of Sturm-Liouville Operators* (Naukova Dumka, Kiev, 1972, in Russian)Google Scholar

39. 39.

V.A. Marchenko, *Sturm−Liouville Operators and Applications* (AMS Chelsea Publishing, Rhode Island, 1986)CrossRefGoogle Scholar

40. 40.

O.I. Marichev, *Method for Computing Integrals of Special Functions* (Nauka i Technika, Minsk, 1978, in Russian)Google Scholar

41. 41.

A.M. Mathai, R.K Saxena, *The H-Function with Applications in Statistics and Other Disciplines* (Halsted Press, Wiley, New York, 1978)Google Scholar

42. 42.

A.P. Prudnikov, Yu. A. Brychkov, O.I. Marichev. *Integrals and Series. More Special Functions*, vol. 3 (Gordon and Breach, New York, 1990)Google Scholar

43. 43.

S.G. Samko, A.A. Kilbas, *Fractional Integrals and Derivatives. Theory and Applications* (Gordon and Breach, Yverdon, 1993), 1112 pp.Google Scholar

44. 44.

S.G. Samko, A.A. Kilbas, O.I. Marichev, *Fractional Integrals and Derivatives: Theory and Applications* (Gordon and Breach Science Publishers, London, 1993)zbMATHGoogle Scholar

45. 45.

C. Shadan, P. Sabatier, *Inverse Problems in Quantum Scattering Theory* (Springer, Berlin, 1989)Google Scholar

46. 46.

S.M. Sitnik, *Unitary and Bounded Buschman–Erdélyi Operators*. Preprint, Institute of Automation and Process Control of the Soviet Academy of Sciences (1990, in Russian), Vladivostok, 44 pp.Google Scholar

47. 47.

S.M. Sitnik, Factorization and estimates of the norms of Buschman-Erdelyi operators in weighted Lebesgue spaces. Soviet Math. Dokladi **44**(2), 641–646 (1992)zbMATHGoogle Scholar

48. 48.

S.M. Sitnik, Transmutations and applications, in *Contemporary Studies in Mathematical Analysis*, Vladikavkaz, ed. by Yu. F. Korobeinik, A.G. Kusraev (2008, in Russian), pp. 226–293Google Scholar

49. 49.

S.M. Sitnik, Factorization method for transmutations in the theory of differential equations. Vestnik Samarskogo Gosuniversiteta **67**(8/1), 237–248 (2008, in Russian)Google Scholar

50. 50.

S.M. Sitnik, Transmutations and applications: a survey (2010), arXiv:1012.3741v1, 141 pp.Google Scholar

51. 51.

S.M. Sitnik, A solution to the problem of unitary generalization of Sonine–Poisson transmutations. Belgorod State Univ. Sci. Bull. Math. Phys. **18**, 5 (76), 135–153 (2010, in Russian)Google Scholar

52. 52.

S.M. Sitnik, Boundedness of Buschman–Erdelyi transmutations, in *The Fifth International Conference "Analytical Methods of Analysis and Differential Equations" (AMADE)*. Mathematical Analysis, Belorussian National Academy of Sciences, Institute of Mathematics, Minsk, vol. 1 (2010, in Russian), pp. 120–125Google Scholar

53. 53.

S.M. Sitnik, Buschman-Erdélyi transmutations, classification and applications (2013). arXiv:1304.2114, 67 pp.Google Scholar

54. 54.

S.M. Sitnik, Buschman-Erdélyi transmutations, classification and applications, in *Analytic Methods of Analysis and Differential Equations (AMADE 2012)*, ed. by M.V. Dubatovskaya, S.V. Rogosin (Cambridge Scientific Publishers, Cambridge, 2013), pp. 171–201Google Scholar

55. 55.

S.M. Sitnik, Buschman-Erdélyi transmutations, their classification, main properties and applications. Sci. Bull. Belgorod State Univ. **39**, 60–76 (2015)Google Scholar

56. 56.

S.M. Sitnik, A survey of Buschman-Erdélyi transmutations. Chelyabinsk Phys. Math. J. **1**(4), 63–93 (2016, in Russian)Google Scholar

57. 57.

S.M. Sitnik, E.L. Shishkina, *Method of Transmutations for Differential Equations with Bessel Operators* (Moscow, Fizmatlit, 2019, in Russian), 224 pp.Google Scholar

58. 58.

H.M. Srivastava, K.C. Gupta, S.L. Goyal, *The H-function of One and Two Variables with Applications* (South Asian Publishers, New Delhi, 1982)zbMATHGoogle Scholar

59. 59.

Kh. Triméche, *Transmutation Operators and Mean-Periodic Functions Associated with Differential Operators*. Mathematical Reports, vol. 4, Part 1 (Harwood Academic Publishers, Reading, 1988)Google Scholar

60. 60.

I.N. Vekua, *Generalized Analytic Functions* (Pergamon Press, Oxford 1962)zbMATHGoogle Scholar

61. 61.

N. Virchenko, I. Fedotova, *Generalized Associated Legendre Functions and Their Applications* (World Scientific, Singapore, 2001)CrossRefGoogle Scholar

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