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INSTRUMENTAL IDENTIFICATION OF STOCHASTIC PROCESS CHARACTERISTICS

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An overview of the most practical and rational methods of instrumental identification of each characteristic of stochastic process is presented. Mathematical descriptions for each method are given and the functional diagrams of the devices are shown.

Introduction. The measurements of the probabilistic characteristics of stochastic processes have spread in various fields of science and technology, and the range of specialists interested in such measurements has expanded considerably. A lot of new measuring techniques and measuring devices have been designed; a lot of journal articles stating the methods and diagrams of devices and the solutions to the theory of measuring problems have appeared.

Today it is quite problematic to cover the whole set of measuring methods and instrumentation, as well as academic papers. Therefore, the choice of methods for each parameter should show the *instrumental identification* of *stochastic process data* and demonstrate the whole idea of the stochastic process itself clearly.

For further work in this direction, there is also a selection of proposed methods, for example, microcontrollers or microprocessors. In its turn it will significantly reduce outlay costs for the identification, dimensions of the entire complex of necessary devices and the speed of the operations to be performed.

The goal: to select and describe the methods of instrumental identification of the.

Objectives: describe the essence of the methods of stochastic process characteristics briefly.

Mean value measurements

Measurements of the mean value are performed by discrete methods according to the algorithm [1]

$$m_x^* = \frac{1}{N}\sum_{i=1}^{N} x(iT_0), \quad (1)$$

where, T_0 — is the range of samples of discrete values of the realization $x(T)$; N — is the total number of samples.

An operation description of a variant of a digital device which measures the mean value of a stationary ergodic stochastic process: the implementation voltage $x(t)$ of the stochastic process $X(t)$ under investigation enters an analog-to-digital converter. At the time of the sampling specified by the polling pulse, the implementation voltage is converted into a proportional number of pulses $q_i = cx(iT_0)$ (proportionality factor $c=10^a$).

These pulses are fed to the input of the time selector and can pass through it to the counter only when the "enabling" voltage is applied from the trigger to the input of the selector, which occurs if the trigger is in position 1. For N samples (polls), the counter accumulates a number.

$$B = \sum_{i=1}^{N} q_i = c\sum_{i=1}^{N} x(iT_0) = cNm_x^* \quad (2)$$

The number of samples (measurement duration) is specified by a circuit consisting of a time selector, a frequency divider, a trigger and a pushbutton switch.

According to the expression (2), measured mean value estimation

$$m_x^* = \frac{B}{cN} \quad (3)$$

Since the proportionality coefficient of the analog-to-digital conversion is 10^a, and the number of samples $N = 10^b$, then

$$m_x^* = B \cdot 10^{-(a+b)} \quad (4)$$

Consequently, the meter reading gives an immediate estimate of the average value, with the number *a+b* determining the position of the comma.

Measurement of average power, dispersion, and root-mean-square deviation

The method of quadrature. The average power (the mean square value) of a stationary ergodic stochastic process is given by

$$p_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)dt, \quad (5)$$

if the limit exists.

To measure the average power, it is necessary to obtain the voltage $y(t)=cx^2(t)$, using a device with a quadratic characteristic, and then to perform averaging. In reality the measurement is carried out for a finite time interval *T*.

The average power of the stochastic process can be measured with the help of various wattmeters.

The root-mean-square value of the implementation voltage, which is often of interest in practice, is measured according to theformula

$$U_* = \sqrt{\overline{x^2(t)}} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}, \quad (6)$$

It is known [2] that the root-mean-square (effective) value of the voltage of the complicated shape is measured by an electronic voltmeter with a quadratic detector.

The operation of extracting the square root is embedded in the graduation scale.

Measuring the variance of a stationary ergodic stochastic process defined by expression

$$D_x = \overline{x^2(t)} - [\overline{x(t)}]^2, \quad (7)$$

results in measuring the average power of the centered process *X(t)*, i.e., the average power of the variable component.

Measurements of correlation functions

To measure the correlation functions, special instruments, called correlometers or correlographs, are used.

In accordance with the principle of action, there are correlometers based on the methods of multiplication, summation (subtraction) and squaring, the approximation of the correlation function by the sum of the terms of its expansion in terms of orthogonal functions, sign correlation, mapping of the scattering diagram, and the like.

One must select a correlometer that works by the multiplication method, performing the actions necessary to obtain estimates of the correlation and mutual correlation function of stationary ergodic stochastic processes in accordance with the expressions

$$K_X^*(\tau) = \frac{1}{T}\int_0^T \overset{\circ}{x}(t)\overset{\circ}{x}(t+\tau)dt, \quad (8)$$

$$K_{XY}^*(\tau) = \frac{1}{T}\int_0^T \overset{\circ}{x}(t)\overset{\circ}{y}(t+\tau)dt, \quad (9)$$

i.e., it carries out a relative shift (delay) for a time τ and multiplies two voltages $\overset{\circ}{x}(t)$ and $\overset{\circ}{x}(t+\tau)$ or $\overset{\circ}{y}(t)$ and $\overset{\circ}{x}(t+\tau)$ averages this product for a sufficiently long interval *T*.

*Digital Correlometers. The d*evices of this type presuppose the sampling of time and quantization (at many levels).

Such devices, which allow obtaining the highest accuracy, are specialized electronic computing systems, in which many blocks and nodes, similar to those used in universal digital computers are applied. It should also be noted that the digital correlometers have been chosen with the expectation of improving the performance of this complicated device for determining the characteristics of stochastic processes.

Spectrum analysis

The spectral power density $G_x(f)$ allows us to judge the frequency properties of the stochastic process $X(t)$. It characterizes its intensity at different frequencies or, in other words, the average power per unit frequency band.

The apparatus spectrum is determined using an instrument called the spectrum analyzer. His work can be based on various methods of analysis.

The most known method is called the filtration method, which consists in isolating narrow sections of the spectrum of the process under investigation using a device with a selective amplitude-frequency response. The main element of the analyzer, which makes it possible to implement this method, is a bandpass filter with a narrow bandwidth in comparison with the width of the spectrum.

The power density can be determined by measuring the average power in the known narrow band. In other words, to measure the power spectral density, it is necessary to "cut out" a narrow band of the spectrum of the process under investigation at first, and then perform the same operations done while measuring the average power (the mean square value) of the stationary stochastic process.

The analysis of probability distribution

The method of analysis under consideration and the principles of constructing the apparatus extend mainly to stationary stochastic processes possessing an ergodic property.

The general functional diagrams of the instrument for such measurements are shown in Figure 1.

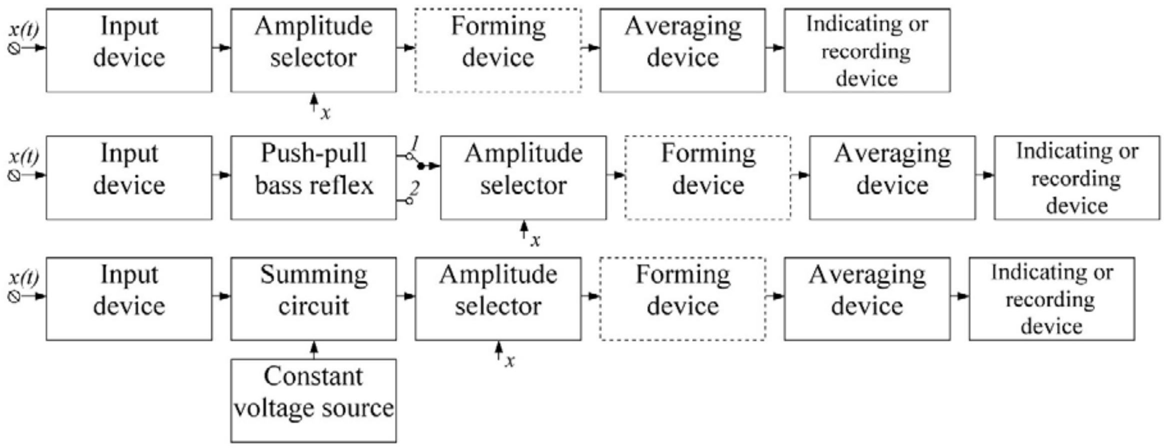

Fig. 1. Functional diagram of probability distribution analyzers

The input device usually has a calibrated attenuator, a cathode (emitter) repeater and, if necessary, an amplifier.

The amplitude selector is a device that emits signals with an amplitude above or below a certain level - the threshold of selection.

The forming device generates single pulses of duration τ_i or Δt_i from the signals received at the output of the amplitude selector. The amplitude selectors, with the combined functions of selection and formation are often used.

The average device is usually an integrator or a low-pass filter.

Magnetoelectric devices, recorders, oscilloscopes with a long afterglow of the screen, digital meters of time intervals are used as indicating and recording devices.

The measurements of conditional probability characteristics

One of the main objectives of the instrumental analysis of stochastic processes is to establish the presence of stochastic relationships between processes and to study the nature and tightness of these interrelationships.

A whole series of objects and processes that have to be dealt with in practice are characterized by nonlinear connections. In such situations, the use of correlation analysis is ineffective and can lead to erroneous conclusions. The regression analysis is more general than the correlation one and is based on measurements of conditional probability characteristics. Such characteristics are the conditional mathematical expectation $M[Y(t)\text{I}x]=m_{Y\text{I}X}$ of one random process relative to the values of the second stochastic process, the function (curve) of regression, the conditional (residual) variance, the variance function of the stochastic process, the mutual dispersion function of two stochastic processes, the correlation ratio of two stochastic processes, the conditional function and the density probability distribution.

Since the listed emission parameters are stochastic variables, theoretical and experimental studies rely on the probabilistic characteristics of these parameters, which are abbreviated as emission characteristics.

The measurement of the probabilistic characteristics of emissions

An analog-to-digital probability distribution analyzer of parameter *V* can be constructed on the basis of a device with a digital discriminator. The addition of this device with a peak detector, a frequency-pulse modulator, a time selector and a delay circuit for rectangular pulses allows us to obtain a universal analyzer for the emission of stochastic processes [3]. Its functional diagram is shown in Figure 2.

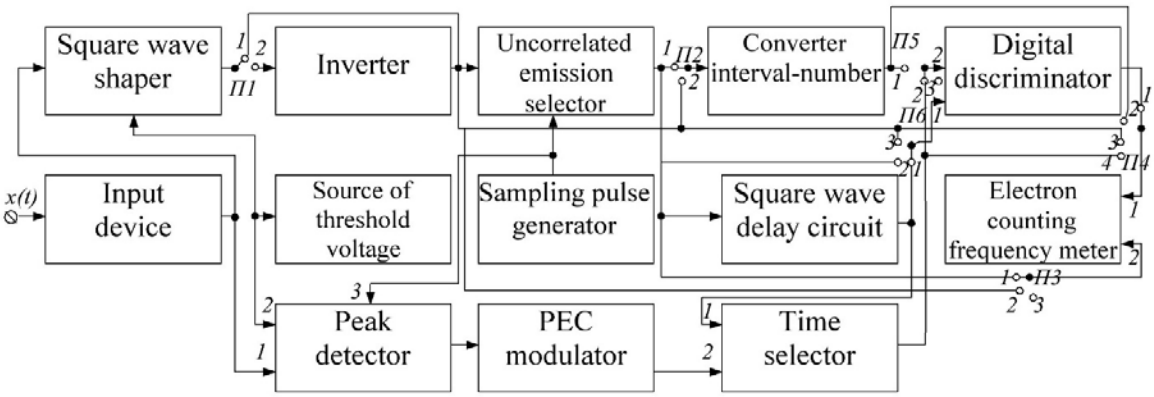

Fig. 2. Universal Emission Analyzer

Conclusion. To summarize it all, we can conclude that while organizing a set of devices for identifying the characteristics of stochastic processes, the selected methods will significantly improve the speed of execution of the necessary calculations, reduce the overall dimensions of the entire complex, thus reducing economic costs. Moreover, it will enable us to perform all the necessary operations using microcontrollers, or microprocessors, while doing further research. This can keep the costs down and result in reducing the size of devices and increasing computational speed.

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