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THE PROBLEM OF ECONOMIC INTERPRETATION OF FRACTIONAL DERIVATIVES

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The paper raises the problem of a possible economic interpretation of one of the mathematical objects that are widely studied today - fractional derivatives (Riemann-Liouville, Caputo, Marchot, conformable fractional derivatives). Particular examples of the interpretation of such derivatives are given.

One of the classical objects for mathematical modeling of dynamic processes (mechanical, physical, economic) is the derivative of a function. For example, the first-order derivative of the function $Y = Y(X)$ by the factor that determines it X sets the limit value corresponding to this indicator. This value describes the growth of the corresponding indicator per unit of growth of the factor that determines it. The derivative of the function describes such economic concepts as marginal utility, elasticity of supply and demand, marginal costs, marginal productivity, marginal cost, marginal income, marginal demand, and others.

In the last 20-30 years in mathematical knowledge, along with classical derivatives, fractional derivatives have begun to play a significant role (in the sense of Riemann-Liouville [1], in the sense of Caputo [1], in the sense of Marchaud [1], conformable fractional derivatives [2] other).

Let $\alpha \in (0,1)$.

Definitions of fractional Derivatives:

Riemann-Liouville derivative [1]:

$$D^{\alpha}(f(x)) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_{-\infty}^{x} (x-\xi)^{-\alpha} f(\xi) d\xi, \quad -\infty < x < +\infty,$$

where Γ= Γ(x) is the gamma-function of Eiler.

Caputo derivative [1]:

$${}_{0}^{C}D_{t}^{\alpha}(f(x)) = \frac{1}{\Gamma(1-\alpha)} \int_{0}^{x} \frac{f'(\xi)}{(x-\xi)^{\alpha}} d\xi, \quad -\infty < x < +\infty.$$

Marchaud derivative [1]

$$D_{+}^{\alpha}(f(x)) = \frac{\alpha}{\Gamma(1-\alpha)} \int_{-\infty}^{x} \frac{f(x) - f(\xi)}{(x-\xi)^{1+\alpha}} d\xi, \quad -\infty < x < +\infty.$$

Conformable fractional derivative [2]

$$T^{\alpha}(f(t)) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \quad t > 0.$$

Many of these derivatives have already received their physical interpretation, and a number of physical processes are actively modeled using the apparatus of fractional integro-differentiation [3]. If we talk about the economic content of such derivatives, then at present there is a significant gap here. Today we are aware of only a few works (see, for example, [4-7]) that allow us to fill it in to some extent. Thus, in the article [7] Tarasova V.V. and Tarasov V.E. the microeconomic meaning of the derivative in the sense of Caputo has been established: it defines the limiting value that describes an economic process with exponential fading memory of the limiting value. In general, according to these authors, fractional derivatives in the sense of Caputo determine [7] economic characteristics (indicators) that are intermediate between the average and marginal indicators.

From the above it follows that today there exists a problem of economic interpretation of fractional derivatives (in the sense of Riemann-Liouville, Marchaud, conformable derivatives), as well as their application in modeling both micro- and macroeconomic processes. A positive solution to this problem in the future will allow scientists, using the apparatus of fractional integro-differentiation, to carry out correct mathematical modeling of economic processes, an adequate description of which previously (using the apparatus of classical mathematical analysis (derivatives of natural order)) was not possible (for example, economic processes related with fading memory of the investigated limiting value).

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