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TWO-DIMENSIONAL INTEGRAL TRANSFORM WITH THE MEIJER G-FUNCTION IN THE KERNEL IN THE SPACE OF SUMMABLE FUNCTIONS

M. PAPKOVICH, O. SKOROMNIK
Polotsk State University, Belarus

Two-dimensional integral transformation with the Meijer G-function in the kernel in the space of summable functions on a domain $R_+^2 = R_+^1 \times R_+^1$ was studied. $\mathfrak{L}_{\overline{\nu},\overline{2}}$ - theory of a considered integral transformation was constructed. Conditions for the boundedness and one-to-one operator of such a transformation from one $\mathfrak{L}_{\overline{\nu},\overline{2}}$ - space to another were given, an analogue of the integration formula in parts was proved, various integral representations for the transformation under consideration were established. The results generalize the well know findings for corresponding one-dimensional integral transformation.

Keywords: *two-dimensional integral G- transformation, Meijer G- function, two-dimensional Mellin transformation, the space of integrable functions, fractional integrals and derivatives.*

Introduction. Let us consider the following integral G-transformation

$$(\mathrm{G}\, f)(\mathrm{x}) = \int_0^\infty \mathrm{G}_{\mathrm{p,q}}^{\mathrm{m,n}}\left[\mathrm{xt}\left|\begin{matrix}(\mathbf{a}_i)_{1,\mathrm{p}}\\(\mathbf{b}_j)_{1,\mathrm{q}}\end{matrix}\right.\right] f(\mathrm{t})d\,\mathrm{t} \quad (\mathrm{x}>0), \tag{1.1}$$

where $\mathrm{x} = (x_1, x_2) \in R^2$; $\mathrm{t} = (t_1, t_2) \in R^2$ - vectors, R^2 - two-dimensional Euclidean space; $\mathrm{x}\cdot\mathrm{t} = \sum_{k=1}^{2} x_k t_k$ – their scalar product, particularly $\mathrm{x}\cdot 1 = \sum_{k=1}^{2} x_k$ for $1 = (1,1)$; $\mathrm{x} > \mathrm{t}$ means $x_1 > t_1, x_2 > t_2$ and similarly for signs $\geq$, $<$, $\leq$; $\int_0^\infty := \int_0^\infty \int_0^\infty$; $N = \{1, 2, \ldots\}$ - space of natural numbers, $N_0 = N \cup \{0\}$, $N_0^2 = N_0 \times N_0$, $R_+^2 = R_+^1 \times R_+^1 = \{\mathrm{x} \in R^2, \mathrm{x} > 0\}$ [1, §28.4];

$\mathrm{m} = (m_1, m_2) \in N_0^2$ и $m_1 = m_2$; $\mathrm{n} = (n_1, n_2) \in N_0^2$ и $n_1 = n_2$;

$\mathrm{p} = (p_1, p_2) \in N_0^2$ и $p_1 = p_2$; $\mathrm{q} = (q_1, q_2) \in N_0^2$ и $q_1 = q_2$; $(0 \leq \mathrm{m} \leq \mathrm{q}, 0 \leq \mathrm{n} \leq \mathrm{p})$;

$\mathbf{a}_i = (a_{i_1}, a_{i_2})$, $1 \leq i \leq \mathrm{p}$, $a_{i_1}, a_{i_2} \in C$ $(1 \leq i_1 \leq p_1, 1 \leq i_2 \leq p_2)$;

$\mathbf{b}_j = (b_{j_1}, b_{j_2})$, $1 \leq j \leq \mathrm{q}$, $b_{j_1}, b_{j_2} \in C$ $(1 \leq j_1 \leq q_1, 1 \leq j_2 \leq q_2)$;

$k = (k_1, k_2) \in N = N \times N$ $(k_1 \in N, k_2 \in N)$ – index with $k! = k_1! k_2!$ and $|k| = k_1 + k_2$; $D^k = \dfrac{\partial^{|k|}}{(\partial x_1)^{k_1} (\partial x_2)^{k_2}}$; $d\,\mathrm{t} = d\,t_1 \cdot d\,t_2$; $f(\mathbf{t}) = (t_1, t_2)$; $\mathrm{G}_{\mathrm{p,q}}^{\mathrm{m,n}}\left[\mathrm{xt}\left|\begin{matrix}(\mathbf{a}_i)_{1,\mathrm{p}}\\(\mathbf{b}_j)_{1,\mathrm{q}}\end{matrix}\right.\right]$ - function such as:

$$\mathrm{G}_{\mathrm{p,q}}^{\mathrm{m,n}}\left[\mathrm{xt}\left|\begin{matrix}(\mathbf{a}_i)_{1,\mathrm{p}}\\(\mathbf{b}_j)_{1,\mathrm{q}}\end{matrix}\right.\right] = \prod_{k=1}^{2} \mathrm{G}_{p_k,q_k}^{m_k,n_k}\left[x_k t_k\left|\begin{matrix}(a_{i_k})_{1,p_k}\\(b_{j_k})_{1,q_k}\end{matrix}\right.\right], \tag{1.2}$$

which is a product of Meijer G- functions $G_{p,q}^{m,n}[z]$ [2, chapter 6].

This paper is devoted to the study of transformation (1.1) in weighted spaces $\mathfrak{L}_{\overline{\nu},\overline{2}}$, $\overline{\nu} = (\nu_1, \nu_2) \in R^2$ $(\nu_1 = \nu_2)$, $\overline{2} = (2,2)$, integrable functions $f(\mathbf{x}) = f(x_1, x_2)$ on R_+^2, for which $\|f\|_{\overline{\nu},\overline{2}} < \infty$, where

$$\|f\|_{\overline{\nu},\overline{2}} = \left\{\int_{R_+^1} x_2^{\nu_2\cdot 2-1}\left[\int_{R_+^1} x_1^{\nu_1\cdot 2-1} |f(x_1,x_2)|^2 dx_1\right] dx_2\right\}^{1/2} < \infty .$$

The conditions of boundedness and mutual uniqueness of the transformation operator (1.1) from one space $\mathfrak{L}_{\overline{\nu},\overline{2}}$ to another are given, an analogue of the integration formula by parts is proved, various integral representations for the transformation under consideration are established. The results obtained generalize those obtained earlier for the corresponding one-dimensional G – transformation [2, ch. 6].

Preliminary information. For integer non-negative m, n, p, q $(0 \le m \le q, 0 \le n \le p)$, for $a_i, b_j \in C$ with *C*, the set of complex numbers $(1 \le i \le p, 1 \le n \le q)$ Meijer *G- function is a function defined via the Mellin-Barnes type integral*:

$$G_{p,q}^{m,n}\left[z\left|\begin{matrix}(a)_p\\(b)_q\end{matrix}\right.\right] = G_{p,q}^{m,n}\left[z\left|\begin{matrix}a_1,...,a_p\\b_1,...,b_q\end{matrix}\right.\right] = G_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_i)_{1,p}\\(b_j)_{1,q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_L \mathcal{G}_{p,q}^{m,n}(s)z^{-s}ds, z \ne 0, \tag{2.1}$$

where

$$\mathcal{G}_{p,q}^{m,n}\left[\begin{matrix}(a_i)_{1,p}\\(b_j)_{1,q}\end{matrix}\left| s\right.\right] = \mathcal{G}_{p,q}^{m,n}\left[\begin{matrix}(a)_p\\(b)_q\end{matrix}\left| s\right.\right] = \frac{\prod_{j=1}^{m}\Gamma(b_j+s)\prod_{i=1}^{n}\Gamma(1-a_i-s)}{\prod_{i=n+1}^{p}\Gamma(a_i+s)\prod_{j=m+1}^{q}\Gamma(1-b_j-s)}. \tag{2.2}$$

Here *L* is specially selected endless contour leaving the poles $s = -b_j - k \ (j = 1,2,...,m;\ k = 0,1,2,...)$ on the left, and the poles $s = 1 - a_j + k \ (j = 1,2,...,n;\ k = 0,1,2,...)$ – on the right, and the empty products, if they occur, are taken to be one. For more details on the theory of the *G*-function (2.1), see [2, ch. 6].

G - transformation is called the integral transformation [2, formula (6.1.1)]

$$(Gf)(x) = \int_0^\infty G_{p,q}^{m,n}\left[xt\left|\begin{matrix}(a_i)_{1,p}\\(b_j)_{1,q}\end{matrix}\right.\right] f(t)dt, \tag{2.3}$$

containing Meijer G - function (2.1) in the kernel.

Introduce the space $\mathfrak{L}_{\nu,r}$ of Lebesgue measurable, generally speaking, complex-valued functions f in $R_+ = (0,\infty)$, for which $\|f\|_{\nu,r} < \infty$, where

$$\|f\|_{\nu,r} = \left(\int_0^\infty \left|t^\nu f(t)\right|^r \frac{dt}{t}\right)^{\frac{1}{r}} \quad (1 \le r < \infty, \nu \in R), \tag{2.4}$$

Notice that

$$\|f\|_{\mathfrak{L}_{\nu,r}} = \|f\|_{L_r(R_+^1, t^{\nu r-1})}, (1 \le r < \infty, \nu \in R).$$

For the function $f \in \mathfrak{L}_{\nu,r}$ $(1 \le r \le 2)$ *Mellin* transformation $\mathfrak{M}f$ is determined by equality [2], [3]

$$(\mathfrak{M}f)(s) = \int_{-\infty}^{+\infty} f(e^\tau)e^{s\tau}d\tau \quad (s = \nu + it; \nu, t \in R). \tag{2.5}$$

If $f \in \mathfrak{L}_{\nu,r} \bigcap \mathfrak{L}_{\nu,1}$, $\operatorname{Re}(s) = \nu$, then (2.5) coincides with the usual Mellin transformation:

$$(\mathfrak{M}f)(s) = f^*(s) = \int_0^{+\infty} f(t)t^{s-1}dt. \tag{2.6}$$

Two-dimensional Mellin transformation of the function $f(\mathrm{x}) = f(x_1, x_2)$, $x_1 > 0, x_2 > 0$, is defined by formula [3, formula1.4.42]:

$$(\mathfrak{M}f)(\mathbf{s}) = f^*(\mathbf{s}) = \int_{R_{++}^2} f(\mathrm{t})\mathrm{t}^{\mathbf{s}-1}d\mathrm{t}, \tag{2.7}$$

$R_{++}^2 = \left\{\mathbf{t} = (t_1, t_2) \in R^2 : t_j > 0 \ (j = 1,2)\right\}$, $\mathbf{s} = (s_1, s_2), s_j \in C \ (j = 1,2)$.

Inverse Mellin transformation for $\mathrm{x} = (x_1, x_2) \in R_{++}^2$ is given by formula [3, formula (1.4.43)]:

$$(\mathfrak{M}^{-1}g)(\mathrm{x}) = \mathfrak{M}^{-1}\left[g(\mathbf{s})\right](\mathrm{x})\frac{1}{(2\pi i)^2}\int_{\gamma_1-i\infty}^{\gamma_1+i\infty}\int_{\gamma_2-i\infty}^{\gamma_2+i\infty} \mathrm{x}^{-\mathbf{s}} g(\mathbf{s})d\mathbf{s}, \ \gamma_j = \operatorname{Re}(s_j) \ (j = 1,2). \tag{2.8}$$

The formula of Mellin transformation from G - transformation (2.3) for "enough good" functions *f* has the form [1, (6.1.2)]

$$(\mathfrak{M}\; Gf)(s) = \mathcal{G}_{p,q}^{m,n}\left[\begin{array}{c|c}(a_i)_{1,p} & \\ & s \\ (b_j)_{1,q} & \end{array}\right](\mathfrak{M}f)(1-s), \qquad (2.9)$$

where $\mathcal{G}_{p,q}^{m,n}(s)$ are given (2.2).

We will need the following constants defined through the parameters of the G-function (2.1) [1, formulas (6.1.5) – (6.1.11)]:

$$\alpha = \begin{cases} -\min\limits_{1\le j\le m}[\mathrm{Re}(b_j)], & m>0, \\ -\infty, & m=0, \end{cases} \qquad \beta = \begin{cases} 1-\max\limits_{1\le i\le n}[\mathrm{Re}(a_i)], & n>0, \\ \infty, & n=0; \end{cases} \qquad (2.10)$$

$$a^* = 2(m+n) - p - q; \qquad (2.11)$$

$$\Delta = q - p; \qquad (2.12)$$

$$a_1^* = m+n-p; \; a_2^* = m+n-q; \qquad (2.13)$$

$$\mu = \sum_{j=1}^{q} b_j - \sum_{i=1}^{p} a_i + \frac{p-q}{2}. \qquad (2.14)$$

We call the exceptional set of $\mathcal{E}_{\mathcal{G}}$ the set $\mathcal{G}(s)$, is defined in (2.2), of real numbers ν such that $\alpha < 1-\nu < \beta$ and $\mathcal{G}(s)$ has a zero on the line $\mathrm{Re}(s) = 1-\nu$.

$\mathfrak{L}_{\overline{\nu},\overline{2}}$ - теория *G*- преобразования

Introduce a two-dimensional analogue of the function (2.2):

$$\overline{\mathcal{G}}_{\mathrm{p,q}}^{\mathrm{m,n}}(\mathbf{s}) \equiv \overline{\mathcal{G}}_{\mathrm{p,q}}^{\mathrm{m,n}}\left[\begin{array}{c|c}(\mathbf{a}_i)_{1,\mathrm{p}} & \\ & \mathbf{s} \\ (\mathbf{b}_j)_{1,\mathrm{q}} & \end{array}\right] = \prod_{k=1}^{2} \mathcal{G}_{p_k,q_k}^{m_k,n_k}\left[\begin{array}{c|c}(a_{i_k})_{1,p_k} & \\ & s_k \\ (b_{j_k})_{1,q_k} & \end{array}\right]. \qquad (3.1)$$

We call *the exceptional set* of $\mathcal{E}_{\overline{\mathcal{G}}}$ of the function $\overline{\mathcal{G}}_{\mathrm{p,q}}^{\mathrm{m,n}}(\mathbf{s})$ the set of vectors $\overline{\nu} = (\nu_1, \nu_2) \in R^2$ $(\nu_1 = \nu_2)$ such that $\alpha_1 < 1-\nu_1 < \beta_1$, $\alpha_2 < 1-\nu_2 < \beta_2$, and the functions type (2.2) $\mathcal{G}_{p_1,q_1}^{m_1,n_1}(s_1)$, $\mathcal{G}_{p_2,q_2}^{m_2,n_2}(s_2)$ have a zero on the line $\mathrm{Re}(s_1) = 1-\nu_1$, $\mathrm{Re}(s_2) = 1-\nu_2$, respectively.

Apply two-dimensional Mellin transformation (2.7) to G - transformation (1.1) and taking into account (2.9), we get the following formula for "enough good" functions *f*:

$$\left(\mathfrak{M}\,\mathrm{G}\,f\right)(\mathbf{s}) = \overline{\mathcal{G}}_{\mathrm{p,q}}^{\mathrm{m,n}}\left[\begin{array}{c|c}(\mathbf{a}_i)_{1,\mathrm{p}} & \\ & \mathbf{s} \\ (\mathbf{b}_j)_{1,\mathrm{q}} & \end{array}\right](\mathfrak{M}f)(1-\mathbf{s}), \qquad (3.2)$$

where $\overline{\mathcal{G}}_{\mathrm{p,q}}^{\mathrm{m,n}}(\mathbf{s})$ is given (3.1).

To formulate statements representing the $\mathfrak{L}_{\overline{\nu},\overline{2}}$ theory of G-transformation (1.1) we need the following two-dimensional analogues of constants (2.10) – (2.14):

$$\alpha_1 = \begin{cases} -\min\limits_{1\le j_1\le m_1}\left[\mathrm{Re}(b_{j_1})\right], & m_1>0, \\ -\infty, & m_1=0, \end{cases}, \quad \beta_1 = \begin{cases} 1-\max\limits_{1\le i_1\le n_1}\left[\mathrm{Re}(a_{i_1})\right], & n_1>0, \\ \infty, & n_1=0, \end{cases} \qquad (3.3)$$

$$\alpha_2 = \begin{cases} -\min\limits_{1\le j_2\le m_2}\left[\mathrm{Re}(b_{j_2})\right], & m_2>0, \\ -\infty, & m_2=0, \end{cases}, \quad \beta_2 = \begin{cases} 1-\max\limits_{1\le i_2\le n_2}\left[\mathrm{Re}(a_{i_2})\right], & n_2>0, \\ \infty, & n_2=0, \end{cases} \qquad (3.4)$$

$$a_1^* = 2(m_1+n_1) - p_1 - q_1,\; a_2^* = 2(m_2-n_2) - p_2 - q_2; \qquad (3.5)$$

$$\Delta_1 = q_1 - p_1, \Delta_2 = q_2 - p_2; \; (18) \qquad (3.6)$$

$$\mu_1 = \sum_{j=1}^{q_1} b_{j_1} - \sum_{i=1}^{p_1} a_{i_1} + \frac{p_1-q_1}{2}, \quad \mu_2 = \sum_{j=1}^{q_1} b_{j_2} - \sum_{i=1}^{p_1} a_{i_2} + \frac{p_2-q_2}{2}, \; (21) \qquad (3.7)$$

Denote by $[X,Y]$ the set of bounded linear operators acting from a Banach space X into a Banach space Y.

From [2, theorem 3.6 and 3.7, theorems 6.1 and 6.2, corollary 6.1.1 and 6.2.1], representation (3.2) and direct verification we get the $\mathfrak{L}_{\overline{\nu},\overline{2}}$ - theory of G- transformation (1.1).

Theorem 1. *We suppose that*

$$\alpha_1 < 1-\nu_1 < \beta_1,\ \alpha_2 < 1-\nu_2 < \beta_2,\ \nu_1 = \nu_2, \tag{3.8}$$

and either of the conditions:

$$a_1^* > 0, a_2^* > 0 \tag{3.9}$$

or

$$a_1^* = 0, a_2^* = 0, \quad \Delta_1[1-\nu_1] + \mathrm{Re}(\mu_1) \le 0, \quad \Delta_2[1-\nu_2] + \mathrm{Re}(\mu_2) \le 0. \tag{3.10}$$

holds.

Then we have the results:

a) There is a one-to-one transformation $\mathrm{G} \in [\mathfrak{L}_{\overline{\nu},\overline{2}}, \mathfrak{L}_{1-\overline{\nu},\overline{2}}]$ *such that (*3.2) *holds for* $f \in \mathfrak{L}_{\overline{\nu},\overline{2}}$ *and* $\mathrm{Re}(\mathbf{s}) = 1-\overline{\nu}$.

If $a_1^* = 0,\ a_2^* = 0,\quad \Delta_1[1-\nu_1] + \mathrm{Re}(\mu_1) = 0,\quad \Delta_2[1-\nu_2] + \mathrm{Re}(\mu_2) = 0$ *u* $\overline{\nu} \notin \mathcal{E}_{\overline{G}}$, *then the transformation* G *maps* $\mathfrak{L}_{\overline{\nu},\overline{2}}$ *onto* $\mathfrak{L}_{1-\overline{\nu},\overline{2}}$.

b) If $f \in \mathfrak{L}_{\overline{\nu},\overline{2}}$ *u* $g \in \mathfrak{L}_{\overline{\nu},\overline{2}}$*,then the relation holds*

$$\int_0^\infty f(\mathrm{x})(\mathrm{G}\,g)(\mathrm{x})d\,\mathrm{x} = \int_0^\infty (\mathrm{G}\,f)(\mathrm{x})g(\mathrm{x})d\,\mathrm{x}. \tag{3.11}$$

c) Let $\overline{\lambda} = (\lambda_1,\ \lambda_1) \in C^2$ *and* $f \in \mathfrak{L}_{\overline{\nu},\overline{2}}$. *If* $\mathrm{Re}(\overline{\lambda}) > -\overline{\nu}$, *then the transformation (1.1) is given by*

$$(\mathrm{G}\,f)(\mathrm{x}) = \mathrm{x}^{-\overline{\lambda}} \frac{d}{d\,\mathrm{x}} \mathrm{x}^{\overline{\lambda}+1} \int_0^\infty G^{\mathrm{m,n+1}}_{\mathrm{p+1,q+1}}\left[\mathrm{xt}\left| \begin{matrix} -\overline{\lambda}, (\mathbf{a}_i)_{1,\mathrm{p}} \\ (\mathbf{b}_j)_{1,\mathrm{q}}, -\overline{\lambda}-1 \end{matrix}\right.\right] f(\mathrm{t})d\,\mathrm{t},\ (26) \tag{3.12}$$

when $\mathrm{Re}(\overline{\lambda}) < -\overline{\nu}$ *is given by*

$$(\mathrm{G}\,f)(\mathrm{x}) = -\mathrm{x}^{-\overline{\lambda}} \frac{d}{d\,\mathrm{x}} \mathrm{x}^{\overline{\lambda}+1} \int_0^\infty G^{\mathrm{m+1,n}}_{\mathrm{p+1,q+1}}\left[\mathrm{xt}\left| \begin{matrix} (\mathbf{a}_i)_{1,\mathrm{p}}, -\overline{\lambda} \\ -\overline{\lambda}-1, (\mathbf{b}_j)_{1,\mathrm{q}} \end{matrix}\right.\right] f(\mathrm{t})d\,\mathrm{t}. \tag{3.13}$$

d) The transformation $\mathrm{G}\,f$ *is independent of* $\overline{\nu}$ *in the sense that, if* $\overline{\nu}$ *and* $\tilde{\overline{\nu}}$ *satisfy* (3.8) *and either (3.9) or (3.10), and if the transformations* $\mathrm{G}\,f$ *and* $\tilde{\mathrm{G}}\,f$ *are defined in* $\mathfrak{L}_{\overline{\nu},\overline{2}}$ *and* $\mathfrak{L}_{\tilde{\overline{\nu}},\overline{2}}$*,respectively, by* (3.2), *then* $\mathrm{G}\,f = \tilde{\mathrm{G}}\,f$ *for* $f \in \mathfrak{L}_{\tilde{\overline{\nu}},\overline{2}} \cap \mathfrak{L}_{\overline{\nu},\overline{2}}$.

e) If $a_1^* > 0, a_2^* > 0$ *or if* $a_1^* = 0, a_2^* = 0,\ \Delta_1[1-\nu_1] + \mathrm{Re}(\mu_1) < 0,\ \Delta_2[1-\nu_2] + \mathrm{Re}(\mu_2) < 0$, *then* $\mathrm{G}\,f$ *is given in (1.1) for* $f \in \mathfrak{L}_{\overline{\nu},\overline{2}}$.

Corollary 1. Let $\alpha_1 < \beta_1, \alpha_2 < \beta_2$ *and let one of the following conditions hold:*

a) $a_1^* > 0, a_2^* > 0$;

b) $a_1^* = 0, a_2^* = 0, \Delta_1 > 0, \Delta_2 > 0$ *u* $\alpha_1 < -\dfrac{\mathrm{Re}(\mu_1)}{\Delta_1}, \alpha_2 < -\dfrac{\mathrm{Re}(\mu_2)}{\Delta_2}$;

c) $a_1^* = 0, a_2^* = 0, \Delta_1 < 0, \Delta_2 < 0$ *u* $\beta_1 > -\dfrac{\mathrm{Re}(\mu_1)}{\Delta_1}, \beta_2 > -\dfrac{\mathrm{Re}(\mu_2)}{\Delta_2}$;

d) $a_1^* = 0, a_2^* = 0, \Delta_1 = 0, \Delta_2 = 0$ *u* $\mathrm{Re}(\mu_1) \le 0, \mathrm{Re}(\mu_2) \le 0$.

Then the G-transformation can be defined on $\mathfrak{L}_{\overline{\nu},\overline{2}}$ *with* $\alpha_1 < \nu_1 < \beta_1, \alpha_1 < \nu_2 < \beta_1,\ \nu_1 = \nu_2$.

Theorem 2. *Let*

$$\alpha_1 < 1-\nu_1 < \beta_1,\ \alpha_2 < 1-\nu_2 < \beta_2,\ \nu_1 = \nu_2,$$

and either of the following conditions holds :

a) $a_1^* > 0, a_2^* > 0$;

b) $a_1^* = 0,\ a_2^* = 0,\quad \Delta_1(1-\nu_1) + \mathrm{Re}(\mu_1) < 0,\quad \Delta_2(1-\nu_2) + \mathrm{Re}(\mu_2) < 0$.

Then for $f \in \mathfrak{L}_{\overline{\nu},\overline{2}}$ *and* $x > 0$ $(\mathrm{G} f)(x)$ *is given in (1.1)*

Corollary 2. *Let* $\alpha_1 < \beta_1, \alpha_2 < \beta_2$ *and let one of the following conditions hold:*

a) $a_1^* > 0, a_2^* > 0$;

b) $a_1^* = 0, a_2^* = 0, \Delta_1 > 0, \Delta_2 > 0$ *u* $\alpha_1 < -\dfrac{\mathrm{Re}(\mu_1)+1}{\Delta_1}, \alpha_2 < -\dfrac{\mathrm{Re}(\mu_2)+1}{\Delta_2}$ *;*

c) $a_1^* = 0, a_2^* = 0, \Delta_1 < 0, \Delta_2 < 0$ *u* $\beta_1 > -\dfrac{\mathrm{Re}(\mu_1)+1}{\Delta_1}, \beta_2 > -\dfrac{\mathrm{Re}(\mu_2)+1}{\Delta_2}$ *;*

d) $a_1^* = 0, a_2^* = 0, \Delta_1 = 0, \Delta_2 = 0$ *u* $\mathrm{Re}(\mu_1) \le 0, \mathrm{Re}(\mu_2) \le 0$.

Then G-transformation can be defined by (1.1) in $\mathfrak{L}_{\overline{\nu},\overline{2}}$ *with* $\alpha_1 < \nu_1 < \beta_1, \alpha_1 < \nu_2 < \beta_1$ *,* $\nu_1 = \nu_2$.

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