UDC 517.926+517.977

CRITERION OF UNIFORM FULL CONTROLLABILITY FOR LINEAR DISCRETE SYSTEMS WITH VARIABLE STRUCTURE

A. KOZLOV, T. ALEKSANDROVICH
Polotsk State University, Belarus

Introduction. One of the actively developing sections of the theory of dynamical systems today is the theory of control of the asymptotic characteristics of linear dynamic (discrete and continuous) systems [1]. The main effective tools used in it and appearing initially in the theory of control of finite-dimensional linear dynamic systems [2] were the controllability matrix (Kalman matrix), as well as the property of uniform complete controllability of a linear controllable differential system. The aim of this work is to introduce the properties of uniform full controllability for linear controlled discrete systems with a varying structure and to obtain a coefficient criterion for the presence of such a property in these systems based on the controllability matrix. The results obtained are planned to be used in future in solving the problems of controlling the asymptotic characteristics of the above-mentioned discrete systems.

Materials and methods. In the present work, the object of study is linear controlled discrete systems with a changing structure, for which the property of their uniform complete controllability is introduced and studied. In the study, methods of the theory of matrices, the theory of discrete dynamic systems, and also the theory of control of linear dynamic systems are used.

Results and its discussion. In case $n_0, n_1, ..., n_t, ...$ and $r_0, r_1, ..., r_t, ...$ two sequences of positive integers, the linear discrete equation is considered to be

$$x_{t+1} = A_t x_t + B_t u_t, \quad t = 0, 1, 2, ..., \tag{1}$$

in which A_t – $(n_{t+1} \times n_t)$ - matrices, B_t – $(n_{t+1} \times r_t)$ - matrices, and sequence u_t at every moment of time takes values in space $\mathbb{R}^{r_t}$ (it plays the role of input). Equation (1) relates to the unknown sequence $x_t \in \mathbb{R}^{n_t}$ at points t and $t+1$.

Definition 1 [3]. The relation (1) for $u_t \equiv 0,\ \ t = 0, 1, 2, ...$, that is, a system of the form

$$x_{t+1} = A_t x_t, \quad t = 0, 1, 2, ...,$$

is called *a homogeneous system with a changing structure.*

The $(n_t \times n_\tau)$ - matrix (Cauchy matrix [3]) $X_{t,\tau}$ is defined in the following way:

$$X_{\tau,\tau} = I_\tau \ \ (I_\tau \text{ – identity transformation into } \mathbb{R}^{n_\tau});$$

$$X_{t,\tau} = A_{t-1} A_{t-2} \cdot \ldots \cdot A_\tau \text{ at } t > \tau.$$

By analogy with [4], the concept of uniform complete controllability of system (1) is introduced.

Definition 2. System (1) is called σ - *uniformly and completely controllable system,* if such numbers as $\sigma \in \mathbb{N}$ and $\gamma > 0$, exist, which for any initial point in time $\tau \in \mathbb{N}$ and in any initial state $x_0 \in \mathbb{R}^{n_\tau}$ there is control $u = u(t),\ \ t = \tau, \tau+1, \ldots, \tau+\sigma-1$, satisfying inequality $\| u(t) \| \leqslant \gamma \| x_\tau \|$ for all $t = \tau, \tau+1, \ldots, \tau+\sigma-1$, and as such the solution of system (1) with this control and the initial condition $x_\tau = x_0$ satisfies equality $x_\sigma = 0 \in \mathbb{R}^{n_{\tau+\sigma}}$.

The controllability matrix is considered [3]

$$W_{\tau,\tau+\sigma} = \sum\nolimits_{j=\tau}^{\tau+\sigma-1} X_{\tau+\sigma, j+1} B_j B_j^T X_{\tau+\sigma, j+1}^T$$

with sizes $n_{\tau+\sigma} \times n_{\tau+\sigma}$.

Theorem. *System (1) is uniformly and completely controllable if and only if such numbers as* $\sigma \in \mathbb{N}$ *and* $\alpha > 0,$ *exist, which for any number* $\tau \in \mathbb{N}$ *for every vector* $\xi \in \mathbb{R}^{n_{\tau+\sigma}}$ *inequality holds*

$$\xi^T W_{\tau,\tau+\sigma}\xi = \xi^T \cdot (\sum_{j=\tau}^{\tau+\sigma-1} X_{\tau+\sigma,j+1} B_j B_j^T X_{\tau+\sigma,j+1}^T) \cdot \xi \geqslant \alpha \| \xi \|^2 .$$

Conclusion. The results presented above will allow us to solve the problem of controlling the asymptotic characteristics of linear discrete systems with a varying structure.

The work was carried out in the framework of the State program of scientific research "Convergence-2020" (subprogram 1, task 1.2.01).

REFERENCES

1. Makarov E.K., Popova S.N. Controllability of asymptotic invariants of non-stationary linear systems. Minsk: Belarus. Navuka, 2012.
2. Kalman R., Falb P., Arbib M. Essays on the mathematical theory of systems. M., 2004
3. Gaishun I.V. Linear systems with a varying structure. Controllability and observability // Differential equations. - 2000. - T. 36, No. 11. - C. 1544-1549.
4. Tonkov E. L. Criterion for uniform controllability and stabilization of a linear recurrent system // Differential equations. - 1979. - T. 15, No. 10. - S. 1804–1813.