UDC 512.7

ON QUESTION ABOUT THE INTERPOLATION POLYNOMIAL OF THE FUNCTION f(λ)

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Let different argument values be given λ_1, λ_2, ..., λ_s and function values $f(\lambda_k)$, $f'(\lambda_k)$, ..., $f^{(m_k-1)}(\lambda_k)$ real or complex variable functions and its derivatives up to and including order (m_k-1) with the specified argument values, where $k=1$, 2, ..., s.

There is a polynomial $p(\lambda)$ of degree $(n-1)$, where $n=m_1+m_2+...+m_s$. This polynomial satisfies the conditions:

$$p(\lambda_k)=f(\lambda_k),\quad p'(\lambda_k)=f'(\lambda_k),\quad p^{(m_k-1)}(\lambda_k)=f^{(m_k-1)}(\lambda_k). \tag{1}$$

The polynomial $p(\lambda)$ is called the Lagrange-Sylvester interpolation polynomial of a function $f(\lambda)$ under interpolation conditions (1).

To solve the problem of interpolating a function $f(\lambda)$ by a polynomial $p(\lambda)$ we find the defining polynomial

$$\psi(\lambda)=(\lambda-\lambda_1)^{m_1}\cdot(\lambda-\lambda_2)^{m_2}\cdot...\cdot(\lambda-\lambda_s)^{m_s}, \text{ where } n=m_1+m_2+...+m_s. \tag{2}$$

Next, we build the right rational fraction

$$\frac{p(\lambda)}{\psi(\lambda)}=\frac{p(\lambda)}{(\lambda-\lambda_1)^{m_1}\cdot(\lambda-\lambda_2)^{m_2}\cdot...\cdot(\lambda-\lambda_s)^{m_s}},$$

which decomposes into a sum of elementary fractions

$$\frac{p(\lambda)}{\psi(\lambda)}=\sum_{k=1}^{s}\left[\frac{\alpha_{k1}}{(\lambda-\lambda_k)^{m_k}}+\frac{\alpha_{k2}}{(\lambda-\lambda_k)^{m_{k-1}}}+...+\frac{\alpha_{km_k}}{(\lambda-\lambda_k)}\right]. \tag{3}$$

To determine the coefficients α_{ij} of the presented decomposition multiply both parts by $(\lambda-\lambda_k)^{m_k}$, then we get

$$\frac{p(\lambda)}{\psi_k(\lambda)}, \text{ where } \psi_k(\lambda)=\frac{\psi(\lambda)}{(\lambda-\lambda_k)^{m_k}}. \tag{4}$$

Substituting $\lambda=\lambda_k$ into a $\frac{p(\lambda)}{\psi_k(\lambda)}$ and into a $\left[\frac{p(\lambda)}{\psi_k(\lambda)}\right]^{(j-1)}$, where $j=1$, 2, ..., m_k,

we find

$$\alpha_{kj}=\frac{1}{(j-1)!}\left[\frac{f(\lambda)}{\psi_k(\lambda)}\right]^{(j-1)}_{\lambda=\lambda_k}. \tag{5}$$

From equality (3) multiplying both parts by $\psi(\lambda)$, we uniquely define a polynomial $p(\lambda)$:

$$p(\lambda)=\sum_{k=1}^{s}\left[\alpha_{k1}+\alpha_{k2}(\lambda-\lambda_k)+...+\alpha_{km_k}(\lambda-\lambda_k)^{m_{k-1}}\right]\cdot\psi_k(\lambda). \tag{6}$$

The expression in square brackets is the sum of the first members m_k of the Taylor series.

Consider a special case of a function $f(\lambda)$: $f(\lambda)=e^{\lambda}$.

We construct an interpolation polynomial $p(\lambda)$ under the following conditions:

$\lambda_1=1$, $\lambda_2=2$, $\lambda_3=3$,

$p(1)=e$, $p(2)=e^2$, $p(3)=e^3$.

First of all, we compose the defining polynomial

$\psi(\lambda)=(\lambda-1)^{m_1}\cdot(\lambda-2)^{m_2}\cdot(\lambda-3)^{m_3}$,

and define

$m_1=2$, $m_2=1$, $m_3=1$.

Then

$n=m_1+m_2+m_3=2+1+1=4$.

Therefore, we seek a polynomial of degree

$n-1=4-1=3$.

$$p(\lambda)=\left(\frac{e}{2}+\frac{5e}{4}(\lambda-1)\right)\cdot(\lambda-2)\cdot(\lambda-3)-e^2(\lambda-1)^2\cdot(\lambda-3)+\frac{e^3}{4}(\lambda-1)^2\cdot(\lambda-2)=$$

$$=\left(\frac{e}{2}+\frac{5e}{4}(\lambda-1)\right)(\lambda^2-3\lambda-2\lambda+6)-e^2(\lambda^2+1-2\lambda)\cdot(\lambda-3)+\frac{e^3}{4}(\lambda^2+1-2\lambda)\cdot(\lambda-2)=$$

$$=\left(\frac{e}{2}+\frac{5e\lambda}{4}-\frac{5e}{4}\right)(\lambda^2-5\lambda+6)-e^2(\lambda^3-3\lambda^2+\lambda-3-2\lambda^2+6\lambda)+\frac{e^3}{4}(\lambda^3-2\lambda^2+\lambda-2-2\lambda^2+4\lambda)=$$

$$=\left(\frac{5e}{4}-e^2+\frac{e^3}{4}\right)\lambda^3+(-7e+5e^2-e^3)\lambda^2+\left(\frac{45e}{4}+\frac{5e^3}{4}-7e^2\right)\lambda+\left(-\frac{9e}{2}+3e^2-\frac{e^3}{2}\right).$$

So, the interpolation polynomial $p(\lambda)$ for a function $f(\lambda)=e^{\lambda}$ for $\lambda_1=1$, $\lambda_2=2$, $\lambda_3=3$, $p(1)=e$, $p(2)=e^2$, $p(3)=e^3$, $p'(1)=e$, $p'(2)=e^2$, $p'(3)=e^3$ has the form:

$$p(\lambda)=\left(\frac{5e}{4}-e^2+\frac{e^3}{4}\right)\lambda^3+(-7e+5e^2-e^3)\lambda^2+\left(\frac{45e}{4}+\frac{5e^3}{4}-7e^2\right)\lambda+\left(-\frac{9e}{2}+3e^2-\frac{e^3}{2}\right).$$

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