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CONCERNING THE QUESTION OF CALCULATING A BEAM WITH A HINGE-ROD CHAIN

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We consider the calculation of a statically indeterminate combined system consisting of a beam and a hinge-rod chain of a general form for the action of an arbitrary vertical load. The qualitative regularities of the internal forces in the rods of the hinge-rod chain are established. Depending on the scheme of interaction of the chain with the beam, the calculation of combined systems with and without a horizontal reaction support is considered. Finite formulas are obtained for determining the internal forces in such systems.

Among the rod structures encountered in construction practice, a very wide class is formed by combined systems consisting of a stiffener beam and a hinge-rod chain (Fig.1). By static properties, such systems can be either statically determinate or statically indeterminate. Particular examples of the calculation of such systems are given in [1], [2].

There are two options of the design of the hinge-rod chain - arched and suspension types. Depending on the scheme of interaction of the chain with the beam, there are combined systems that have a horizontal reaction support (spread) (Fig. 1.a) and ones that do not (non-spread) (Fig.1.b).

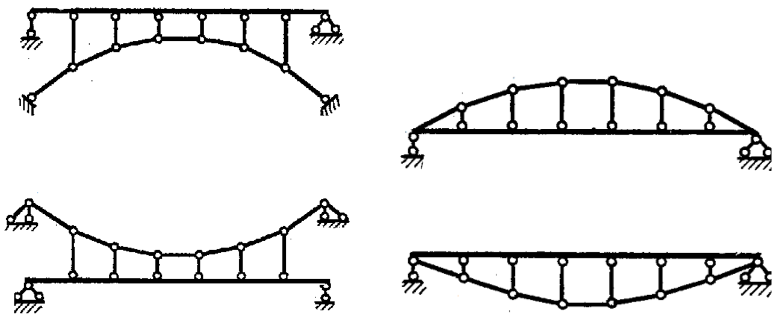

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Figure 1. – Combined systems

With respect to the difference in the interaction schemes of the chain with the beam, it is of interest to consider the qualitative laws of the internal forces arising in the elements of the chain. We assume that the chain has an arbitrary finite number of nodes.

Since the elements of the chain act as truss rods, only longitudinal forces arise in them. Let us consider the longitudinal force diagram in an arbitrary node of the chain of number n (Fig.2).

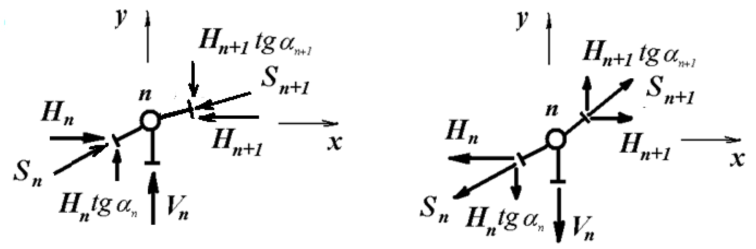

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Figure 2. – Scheme of longitudinal forces in an arbitrary node of the chain

In the case of the arched version, all elements will be compressed (Fig.2.a), and in the case of the suspension version, they will be tensioned (Fig.2.b). For the forces acting on an arbitrary node of the chain, we compose the projection equations on the x and y axes.

From the equation of projections onto the x axis it follows that

$$H_n = H_{n+1} = H = const \quad (n = 1, 2, ...) \tag{1}$$

The resulting relation (1) characterizes the first property of the chain. The horizontal components of the longitudinal forces in all inclined chain rods are the same. This same horizontal component is a chain spread. Then the longitudinal forces in the inclined rods of the chain are related to its spread by the ratio

$$S_n = \frac{H}{\cos\alpha_n} \tag{2}$$

From the equation of projections onto the y axis, it follows that the longitudinal forces in the vertical rods are related to the chain spread by the following relation

$$\left|V_n\right| = H\left(tg\alpha_{n+1} - tg\alpha_n\right) \tag{3}$$

The obtained relations (2), (3) are characterized by the second property of the chain. The internal forces in all chain rods are expressed through the chain spread.

We consider the chain case, that is most common in construction practice, when the distance between the chain posts is the same

$$a_{n-1} = a_n = a_{n+1} = a \tag{4}$$

and the chain nodes are outlined according to the law of a quadratic parabola

$$y = kx^2 \tag{5}$$

In this case, the position of arbitrary three adjacent nodes of the срфшт, taking into account (4), (5), is described by the following relations:

- nodes' abscissas

$$\begin{aligned} &x_{n-1} \\ &x_n = x_{n-1} + a \\ &x_{n+1} = x_{n-1} + 2a \end{aligned} \tag{6}$$

- nodes' ordinates

$$\begin{aligned} &y_{n-1} = kx_{n-1}^2 \\ &y_n = k\left(x_{n-1} + a\right)^2 \\ &y_{n+1} = k\left(x_{n-1} + 2a\right)^2 \end{aligned} \tag{7}$$

Hence, the tangents of the slope angles of the chain rods adjacent to an arbitrary node of the chain of number n, taking into account (6), (7), are related to the corresponding ordinates by the relations

$$\begin{aligned} tg\alpha_n &= \frac{y_n - y_{n-1}}{a} \\ tg\alpha_{n+1} &= \frac{y_{n+1} - y_n}{a} \end{aligned} \tag{8}$$

Substituting (7), (8) into (3), after the corresponding transformations, we obtain that the force in an arbitrary chain's vertical element is described by the expression

$$\left|V_n\right| = 2akH \tag{9}$$

From the resulting expression (9), it follows that the longitudinal forces in all the vertical elements of the chain are the same in magnitude.

Taking into consideration the established properties of the hinge-rod chain, we consider the calculation by the Force Method of the stiffener beam and the hinge-rod chain of a general form for the action of an arbitrary vertical load (Fig.1).

The initial set parameters of combined systems in all cases considered are:

– type of the system: *t* = 1 – suspension; *t* = 2 – arch;
– span value- *l* и chain rise – *f*;

– number of panels - *p*;

– load parameters - places of application and direction of concentrated and distributed loads, values of their modules and intensities;

– stiffness beam parameters — elastic modulus of the material - E_b, area - A_b and moment of inertia - I_b of the cross section;

– chain elements parameters - elastic modulus of the material of the belt rods - E_s, the elastic modulus of the material of the struts (vertical elements) - E_v, the cross-sectional area of the rods of the belt - A_s, the cross-sectional area of the struts - A_v.

The derived parameters of combined systems in all cases considered are:

– belt panel length - $a = \frac{l}{p}$;

– number of nodes in the chain - $k_y = p + 1$;

– number of rods in the belt of the chain - $k_s = p$;

– number of struts in the chian - $k_v = p - 1$.

– The geometry of the hinge-rod chain of the calculated combined systems is characterized by;

– outline law of the chain's axis – $y(x)$;

– coordinates of the chains nodes - $y_k \quad \left(k = 1, ..., k_y\right)$;

– length of the belt's rods - $l_{s_k} = \sqrt{\left(y_{k+1} - y_k\right)^2 + a^2} \quad \left(k = 1, ..., p\right)$;

– angle of the chain rods - $tg\alpha_k = \frac{y_k - y_{k+1}}{a} \quad \left(k = 1, ..., p\right)$;

– length of the struts - $l_{v_k} \quad \left(k = 1, ..., k_v\right)$.

The considered spreader and non-spreader combined systems for both options of the structural design of the hinge-rod chain are once statically indeterminate systems. In all cases, to remove the only excess connection, the hinge cuts the middle section of the beam and, therefore, the main system of the force method is a statically determinate combined system (Fig.3).

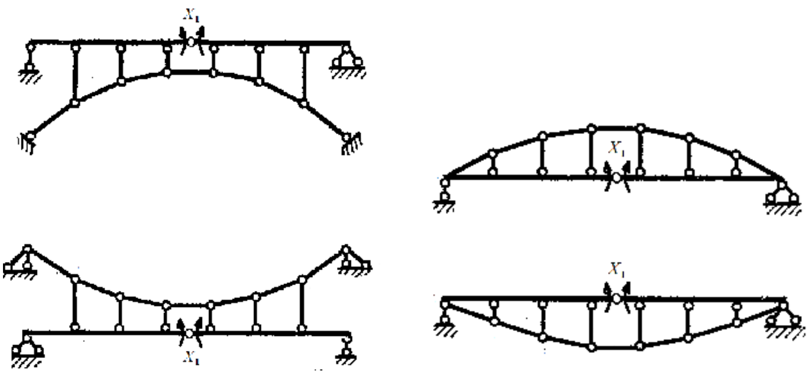

Figure 3. – Main system of the Force Method

The canonical equation of the force method for all considered combined systems has the same form

$$\delta_{11} X_1 + \Delta_{1P} = 0$$

and the main unknown in all cases is the bending moment in the most dangerous section of the stiffener. The final formulas are obtained that relate the internal forces of the combined systems under consideration with the beam internal forces, with the chain spread and the main unknown.

The formulas for determining the internal forces in the hinge-rod chain are:

– chain spread - $H = \frac{M_{Cb} - X_1}{f}$

– longitudinal force in the belt's rods - $S_k = \frac{M_{Cb} - X_1}{f \cos \alpha_k} \quad (k = 1, ..., p)$

– longitudinal force in the struts - $V_k = \frac{M_{Cb} - X_1}{f} (tg\alpha_{k+1} \text{-} tg\alpha_k) \quad (k = 1, ..., k_v)$

The formulas for determining the internal forces in the stiffener beam are:

– non-spread combined systems of any type

$$M(x) = M_b(x) + \frac{X_1 - M_{Cb}}{f} y(x)$$

$$Q(x) = Q_b + \frac{X_1 - M_{Cb}}{f} tg\alpha(x)$$

$$N = \frac{X_1 - M_{Cb}}{f}$$

– spread combined systems of any type

$$M(x) = M_b(x) + \frac{X_1 - M_{Cb}}{f} y(x)$$

$$Q(x) = Q_b + \frac{X_1 - M_{Cb}}{f} tg\alpha(x)$$

$$N = 0$$

The established properties of the hinge-rod chain and the resulting final formulas for determining the internal forces in the considered combined systems are of practical importance and can be used both when designing new and when strengthening existing combined systems consisting of a beam and a hinge-rod chain.

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