should take a responsible approach to vocabulary and bot training, the choice of the strategy and technology for its application in the classroom, think through the methodological and pedagogical meaning of its implementation. That means that working with a dialogue bot requires a very thorough and, as a rule, long preparatory stage, as the functioning of the program largely depends on it.

There may be many variants of technological application of this model in teaching: with the help of a dialogue bot we can test students' knowledge on a given topic using the latest technologies, students can apply to the bot for reference information submitted in an interesting form. After the conversation with the bot, the teacher can review the logs (full recording of the dialogue with the program) and draw a conclusion about the level of vocabulary assimilation on the topic. It is also possible for students to create their own dialogue dictionaries and learn from them as a special way to practice vocabulary.

Conclusion. The current level of technologies development of the world dictates the need for an ever-increasing transformation of the educational process. The use of described in the article prototype makes learning more effective, interactive and interesting for students, what should be given special attention in the development of new paradigms and norms in education today, as well as new understanding and reflection of the learning process itself.

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UNIFORM GLOBAL ATTAINABILITY OF A LINEAR CONTROLLED DISCRETE-TIME PERIODIC SYSTEM

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Today, along with the study of linear controlled periodic systems of ordinary differential equations much attention is paid to the exploration of similar systems with discrete time.

The main aim is to receive the necessary and sufficient conditions of uniform global attainability of a linear discrete-time controlled periodic system.

Material and methods. We obtained the main results using the methods of theory of control over asymptotic invariants of linear differential systems, linear algebra and matrix theory.

Findings and their discussion. Let $\mathbb{R}^n$ be a n-dimensional Euclidean vector space supplied with the norm $\|x\|=\sqrt{x^T x}$ (here the symbol T means the transpose of a matrix or a vector); $e_1, e_2, \ldots, e_n$ be vectors (columns) of the canonical orthonormal basis for the space $\mathbb{R}^n$; M_{mn} be the space of real $m \times n$-dimensional matrices supplied with the spectral (operator) norm $\| H \| = \max\limits_{\|x\|=1} \| Hx \|,$ i.e. the norm induced by the Euclidean norm in the spaces $\mathbb{R}^n$ and $\mathbb{R}^m$ [1, p. 357]; $\mathrm{M}_n := \mathrm{M}_{nn}$. Denote by $E = [e_1, \ldots, e_n] \in \mathrm{M}_n$ the identity matrix. An interval $[t_0,\ t_1)$, where $t_0, t_1 \in \mathbb{Z}$, $t_0 < t_1$, is understood as the set of integer points $t_0, t_0 + 1, \ldots, t_1 - 1$ (respectively $[t_0, +\infty) = \{t_0, t_0 + 1, \ldots\}$).

For a linear time-varying homogeneous discrete system

$$x(t+1) = A(t)x(t),\ t \in \mathbb{Z},\ x \in \mathbb{R}^n \tag{1}$$

denote by $X(t, \tau),\ t \geq \tau,$ the Cauchy matrix, i.e.

$$X(t, \tau) = \begin{cases} A(t-1) \cdot \ldots \cdot A(\tau), & t > \tau, \\ E, & t = \tau. \end{cases}$$

If $\det A(t) \neq 0$ for all $t \in \mathbb{Z}$ then $X(t,\tau)$ is invertible for all $t \geq \tau$. In that case, we set $X(\tau, t) := X^{-1}(t, \tau),\ t > \tau.$ Thus, $X(t,\tau)$ will be defined for all $t, \tau \in \mathbb{Z}$ and

$$X(t, \tau) = A^{-1}(t) \cdot \ldots \cdot A^{-1}(\tau - 1), \quad t < \tau.$$

Consider a linear control discrete-time system with ω-periodic coefficients

$$x(t + 1) = A(t)x(t) + B(t)u(t),\ t \in \mathbb{Z},\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ \omega \in \mathbb{N} \setminus \{0\}. \tag{2}$$

We close this system with a linear feedback $u = U(t)x,$ where U is a bounded and measurable matrix and receive homogeneous system

$$x(t + 1) = (A(t) + B(t)U(t))x,\ x \in \mathbb{R}^n \tag{3}$$

Definition 1. [2, p. 217] The system (2) is said to be *completely controllable on the interval* $[\tau, \tau + \vartheta)$ ($\tau \in \mathbb{Z}$, $\vartheta \in \mathbb{N}$) if for any $x_0, x_1 \in \mathbb{R}^n$ there exists a control $u(t),\ t \in [\tau, \tau + \vartheta),$ that transfers the solution of the system (2) from the point $x(\tau) = x_0$ into the point $x(\tau + \vartheta) = x_1.$

Let $X_U(t,\tau)$ is the Cauchy matrix of the system (3) with the control $U(\cdot).$ Suppose $X(t,\tau), t \geq \tau,$ is the Cauchy matrix of the system (1) (of the system (2) with $u(t) = 0$, $t \in \mathbb{Z}$, i.e. $X(t,\tau) = X_0(t,\tau)$). For $t > \tau$ let us construct the matrix

$$W_1(t, \tau) = \sum_{s=\tau}^{t-1} X(t, s+1) B(s) B^T(s) X^T(t, s+1).$$

Definition 2. [3] The system (2) is said to be ϑ-uniform completely controllable ($\vartheta \in \mathbb{N}$) if there for all $\xi \in \mathbb{R}^n$ exists such $\alpha_i = \alpha_i(\vartheta) > 0,\ i = \overline{1, 4},$ that for all $\tau \in \mathbb{Z}$ the following inequalities hold

$$\xi^T W_1(\tau+\vartheta,\tau)\xi > 0,\ \alpha_1 \|\xi\|^2 \le W_1^{-1}(\tau+\vartheta,\tau) \le \alpha_2 \|\xi\|^2,$$
$$\alpha_3 \|\xi\|^2 \le X^T(\tau+\vartheta,\tau)W_1^{-1}(\tau+\vartheta,\tau)X(\tau+\vartheta,\tau) \le \alpha_4 \|\xi\|^2 .$$

The system (2) is said to be *uniform completely controllable* ($\vartheta \in \mathbb{N}$) if there exists a such $\vartheta > 0$ that the system (2) is ϑ-uniform completely controllable [3].

Definition 3. A system (3) has the property of *uniform global attainability* if there exists $T \in \mathbb{Z}$, $T > 0$, that for any $r > 1$ and $0 < \rho \le 1$ there is $\theta = \theta(r,\rho) > 0,$ at which for an arbitrary matrix $H \in M_n$, $\|H - E\| \le r,\ \det H \ge \rho$ and any $t_0 \in \mathbb{Z}$, there exists a measurable and bounded control $U:[t_0, t_0+T] \to M_{mn}$ satisfying for all $t \in [t_0, t_0+T]$ $\|U(t)\| \le \theta(r,\rho)$ and guaranteeing for the Cauchy matrix $X_U(t,s)$ of the system (3) the equality $X_U(t_0+T, t_0) = H.$

The property of uniform global attainability of the system (3) makes it possible to control the entire finite-dimensional basis of the solution space of this system on an arbitrary time interval of fixed length $T,$ i.e. allows you to choose such a matrix control $U,$ in which the set $\{x_i(t)\}_{i=1}^n$ of a linear independent system (3) with this control and initial conditions (the appropriate vectors $e_i, i=\overline{1,n},$ canonical orthonormal basis of the space $\mathbb{R}^n$) through time T will be equal to an arbitrary predetermined right basis of this space.

This property was studied for stationary systems with constant coefficients

$$\dot{x} = (A + BU(t))x,\ x \in \mathbb{R}^n,\ t \in \mathbb{R} \qquad (4)$$

by V.A. Zaitsev (for $n = 2,3$), by V. A. Zaitsev and by A.F. Gabdrakhimov (for case $n = 4$), and they proved uniform global attainability in the class of piecewise constant controls, by I.N. Sergeyev using the method of reduction to the canonical form a uniform global attainability of a stationary system (4) of arbitrary dimension of the phase space was established under the condition of full controllability of the corresponding system $n = 2,3.$

For a non-stationary linear system

$$\dot{x} = (A(t) + B(t)U(t))x,\ x \in \mathbb{R}^n,\ \mathrm{t} \in \mathbb{R}, \qquad (5)$$

with locally integrable and integrally bounded coefficients, given in a special form-in the lower form of Hessenberg, V.A. Zaitsev obtained sufficient conditions for uniform global attainability [4, p. 258–259] of the appropriate system $\dot{x} = A(t)x,\ x \in \mathbb{R}^n,\ \mathrm{t} \in \mathbb{R},$ Uniform global attainability for systems (5) of general form with piecewise continuous and bounded coefficients was obtained by S.N. Popova and E.K. Makarov [4, p. 310–325], for $n = 2$ and under the assumption of uniform complete controllability of the corresponding system $\dot{x} = A(t)\,x + B(t)u,\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^{\mathrm{m}},\ \mathrm{t} \in \mathbb{R},$ and piecewise uniform continuity [4, p. 264–265] of the matrix $B.$ I.V. Ints proved uniform global attainability of arbitrary two-dimensional systems (5) with locally integrable and integrally bounded coefficients [5]. S.N. Popova obtained weaker condition of uniform global reducibility for discrete-time periodic systems [6]. A.A. Kozlov proved uniform global attainability for linear non-stationary periodic systems with piecewise continuous and bounded coefficients [7].

In this paper necessary and sufficient condition of uniform global attainability of non-stationary discrete-time periodic system, i.e. we proved

Theorem 1. *A linear discrete-time periodic control system* (2) *is uniform global controllable iff the corresponding linear closed system* (3) *has the property of uniform global attainability*.

Conclusion. The obtained results can be further used in the theory of control over asymptotic invariants of dynamic systems of arbitrary types.

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MODELING THE REGIONS OF BELARUS COMPETITIVENESS THE BASED ON PANEL DATA

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Comparative analysis of the regions of Belarus competitiveness was conducted. A system of indicators that reflect the competitiveness in the regions under study was built. It consists of five units: quality of the population, living standards, quality of social services, quality of the ecological niche, cultural condition of society, investment attractiveness. Integral indicator of the competitiveness for regions was built using the factor analysis. All baseline indicators were sorted according to their impact.