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O. V. Skoromnik, M. V. Papkovich

MULTIDIMENSIONAL H-TRANSFORM IN THE WEIGHTED SPACE OF SUMMABLE FUNCTIONS

Multidimensional integral transform

$$(\mathbf{H}f)(\mathbf{x}) = \int\limits_{\mathbf{0}}^{\mathbf{x}} \mathrm{H}_{\mathbf{p},\mathbf{q}}^{\mathbf{m},\mathbf{n}}\left[\mathbf{xt}\,\middle|\, \begin{matrix}(\mathbf{a}_i,\alpha_i)_{1,p}\\ (\mathbf{b}_j,\beta_j)_{1,q}\end{matrix}\right] f(\mathbf{t})\,\mathbf{dt} \quad (\mathbf{x}>0) \tag{1}$$

is studied. Here (see [1, Section 28.4; 2]) $\mathbf{x}=(x_1,x_2,\ldots,x_n) \in \mathbb{R}^n$; $\mathbf{t}=(t_1,t_2,\ldots,t_n)\in \mathbb{R}^n$, where $\mathbb{R}^n$ is Euclidean n-space; $\mathbf{x}\cdot\mathbf{t}=\sum_{n=1}^{n} x_n t_n$ denotes their scalar product; in particular, $\mathbf{x}\cdot\mathbf{1}=\sum_{n=1}^{n} x_n$ for $\mathbf{1}$=(1,...,1). The expression $\mathbf{x}>\mathbf{t}$ means that $x_1>t_1$, $\ldots$, $x_n>t_n$, the nonstrict inequality $\geqslant$ has similar meaning; $\int_{\mathbf{0}}^{\mathbf{x}}=\int_{\mathbf{0}}^{x_1}\int_{\mathbf{0}}^{x_2}\cdots\int_{\mathbf{0}}^{x_n}$; by $\mathbb{N}=\{1,2,\ldots\}$ we denote the set of positive integers, $\mathbb{N}_0=\mathbb{N}\bigcup\{0\}$, $\mathbb{N}_0^n=\mathbb{N}_0\times\mathbb{N}_0\times\ldots\times\mathbb{N}_0$, $\mathbb{R}_+^n=\{\mathbf{x}\in\mathbb{R}^n,\ \mathbf{x}>0\}$;

$\mathbf{m}=(m_1,m_2,\ldots,m_n)\in\mathbb{N}_0^n$ and $m_1=m_2=\ldots=m_n$; $\mathbf{n}=(\overline{n}_1,\overline{n}_2,\ldots,\overline{n}_n)\in\mathbb{N}_0^n$ and $\overline{n}_1=\overline{n}_2=\ldots=\overline{n}_n$; $\mathbf{p}=(p_1,p_2,\ldots,p_n)\in\mathbb{N}_0$ and $p_1=p_2=\ldots=p_n$; $\mathbf{q}=(q_1,q_2,\ldots,q_n)\in\mathbb{N}_0$ and $q_1=q_2=\ldots=q_n)$ $(0\leqslant\mathbf{m}\leqslant\mathbf{q},\ 0\leqslant\mathbf{n}\leqslant\mathbf{p})$; $\mathbf{a}_i=(a_{i_1},a_{i_2},\ldots,a_{i_n})$, $1\leqslant i\leqslant p$, $a_{i_1},a_{i_2},\ldots,a_{i_n}\in\mathbb{C}$ $(1\leqslant i_1\leqslant p_1,\ \ldots,\ 1\leqslant i_n\leqslant p_n)$;

$\mathbf{b}_j=(b_{j_1},b_{j_2},\ldots,b_{j_n})$, $1\leqslant j\leqslant q$, $b_{j_1},b_{j_2},\ldots,b_{j_n}\in\mathbb{C}$ $(1\leqslant j_1\leqslant q_1,\ \ldots,\ 1\leqslant j_n\leqslant q_n)$;

$\alpha_i=(\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_n})$, $1\leqslant i\leqslant p$, $\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_n}\in\mathbb{R}_+^1$ $(1\leqslant i_1\leqslant p_1,\ldots,1\leqslant i_n\leqslant p_n)$;

$\beta_j=(\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n})$, $1\leqslant j\leqslant q$, $\beta_{j_1},\beta_{j_2},\ldots,\beta_{j_n}\in\mathbb{R}_+^1$ $(1\leqslant j_1\leqslant q_1,\ldots,1\leqslant j_n\leqslant q_n)$;

$\mathbf{k}=(k_1,k_2,\ldots,k_n)\in\mathbb{N}_0^n=\mathbb{N}_0\times\ldots\times\mathbb{N}_0$ $(k_i\in\mathbb{N}_0,\ i=1,2,\ldots,n)$ is a multi-index with $\mathbf{k}!=k_1!\cdots k_n!$ and $|\mathbf{k}|=k_1+k_2+\ldots+k_n$; for $l=(l_1,l_2,\ldots,l_n)\in\mathbb{R}_+^n$;

$\mathbf{D}^l=\frac{\partial^{|l|}}{(\partial x_1)^{l_1}\cdots(\partial x_n)^{l_n}}$, $\mathbf{dt}=dt_1\,dt_2\cdots dt_n$; $\mathbf{t}^l=t^{l_1}\cdots t^{l_n}$; we introduce the function

$$\mathrm{H}_{\mathbf{p},\mathbf{q}}^{\mathbf{m},\mathbf{n}}\left[\mathbf{xt}\,\middle|\, \begin{matrix}(\mathbf{a}_i,\alpha_i)_{1,p}\\ (\mathbf{b}_j,\beta_j)_{1,q}\end{matrix}\right] = \prod_{k=1}^{n} \mathrm{H}_{p_k,q_k}^{m_k,\overline{n}_k}\left[x_k t_k\,\middle|\, \begin{matrix}(a_{i_k},\alpha_{i_k})_{1,p_k}\\ (b_{j_k},\beta_{j_k})_{1,q_k}\end{matrix}\right],$$

which is the product of the H-functions $\mathrm{H}_{p,q}^{m,n}[z]$ [3, Chapters 1 and 2]. Our paper is devoted to the study of transform (1) in the weighted spaces $\mathfrak{L}_{\overline{\nu},\overline{2}}$ summable functions $f(\mathbf{x})=f(x_1,\ldots,x_n)$ on $\mathbb{R}_+^{\mathbf{n}}$, such that:

$$\|f\|_{\overline{v},\overline{2}}=\left\{\int\limits_{R_+^1} x_n^{v_n\cdot 2-1}\left\{\cdots\left\{\int\limits_{R_+^1} x_2^{v_2\cdot 2-1}\left[\int\limits_{R_+^1} x_1^{v_1\cdot 2-1}|f(x_1,\ldots,x_n)|^2\,dx_1\right]dx_2\right\}\cdots\right\}dx_n\right\}^{1/2}<\infty,$$

where $\overline{2} = (2, \ldots, 2)$, $\overline{v} = (v_1, \ldots, v_n) \in \mathbb{R}^n$, $v_1 = v_2 = \ldots = v_n$.

In our report we apply results from [3, Chapter 3.6] and [4] to prove the boundedness and one-to-one property of the operator $\mathbf{H}f$ of transform (1) from one space $\mathfrak{L}_{\overline{\nu},\overline{2}}$ to the another, to present various integral representations and characterization of images of these operator, and to establish their inversion formulae. The results presented generalize those obtained in [3, Chapter 3.6] for one-dimensional case.

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O. Tsekhan, E. Pawluszewicz

STABILITY AND STABILIZABILITY OF SINGULARLY PERTURBED SYSTEM ON TIME SCALES ON THE BASIS OF DECOMPOSITION

A *time scale* [1] $\mathbb{T}$ is an arbitrary nonempty closed subset of the set $\mathbb{R}$ of real numbers. As a standard cases $\mathbb{R}$ and $h\mathbb{Z}$, $h > 0$, are time scales. Let $\mathcal{S}(\mathbb{T})$ is the set of exponential stability [1, Definition 3.2] for time scale $\mathbb{T}$.

Consider a singularly perturbed linear time-invariant system (SPLTIS) on time scale $\mathbb{T}$

$$x^{\Delta}(t) = \mu A_1 x(t) + \mu A_2 y(t) + \mu B_1 u(t), \tag{1}$$

$$y^{\Delta}(t) = A_3 x(t) + A_4 y(t) + B_2 u(t), \quad t \in [0, \infty)_{\mathbb{T}}, \tag{2}$$

where $[0, \infty)_{\mathbb{T}} = [0, \infty) \bigcap \mathbb{T}$, μ is a small parameter, $\mu \in (0, \mu_0]$, $\mu_0 \ll 1$, $x \in \mathbb{R}^{n_1}$ is a slow variable, $y \in \mathbb{R}^{n_2}$ is a fast variable, $u \in \mathbb{R}^r$ is a control, $u(t)$ is an element of the set U of bounded rd-continuous functions that don't depend on μ, $x(0) = x_0 \in \mathbb{R}^{n_1}$, $y(0) = y_0 \in \mathbb{R}^{n_2}$, A_i, $i = \overline{1, 4}$, B_1, B_2 are matrices of appropriate dimensions.

Assumption. Time scale $\mathbb{T}$ is unbounded above, $\mathcal{S}(\mathbb{T})$ is open subset in $\mathbb{R}$ and for some $k > 0$ $B_k(-k) \subset \mathcal{S}(\mathbb{T})$, where $B_k(-k)$ denotes the disc with the center at $(-k, 0)$ and the radius of k.

The Assumption is valid, for instance, for homogenious [1] time scales and some another time scales (e.g. Example 3.6 I-V in [1]).

In our report we discuss stability and stabilizability conditions of SPLTIS (1), (2) for all sufficient small $\mu > 0$ defined on time scale $\mathbb{T}$ and satisfying the Assumption. The results presented generalize those obtained in [2] for $\mathbb{T} = \mathbb{R}$ and [3] for $\mathbb{T} = \mathbb{Z}$.

Let det $A_4 \neq 0$. Consider for SPLTIS (1), (2) slow subsystem of the form

$$\bar{x}^{\Delta}(t) = \mu A_0 \bar{x} + \mu B_0 u_s(t), \quad \bar{x} \in \mathbb{R}^{n_1},$$

$$A_0 = A_1 - A_2 A_4^{-1} A_3, \quad B_0 = B_1 - B_2 A_4^{-1} A_3, \quad \bar{x}(0) = x_0, \tag{3}$$