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ON TAKING ACCOUNT OF THE HETEROMODULARITY OF THE MATERIAL WHEN MAKING CALCULATIONS OF TRUSSES WITH RIGID JOINTS

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This article deals with the influence of the heteromodularity of the material on the parameters of the tensity and deformity state (TDS) of truss constructions with a rigid connection of the rods in the joints with a nodal scheme of loading. For the heteromodularity of the material to be taking into account one should use a bilinear diagram of the dependence between tensions and deformations of the material in truss rods. The calculation of trusses is carried out with the help of the force method. The determination of the coefficients and free terms of canonical equations is fulfilled according to Maxwell-More's generalized formula, which takes into account the heteromodularity of the material. The influence of the heteromodularity of the construction material on the parameters of the TDS of a truss is illustrated with a representative example.

In the linear theory of calculating pivoted constructions, the construction material is, as a rule, considered a homogeneous isotropic elastic body the behavior of which is described with the modulus of elasticity *E* which is the same in case of tensility and pressure. However, as the experimental investigations [1, 2] show, the moduli of elasticity at tension E^{+} and pressure E^{-} are different for a number of modern construction materials. They differ essentially.

More than that, the phenomenon of heteromodularity, to this or that extent, is inherent practically in all traditional construction materials. Thus, according to [3], heteromodularity is established for numerous steels and alloys. The coefficient of heteromodularity () for them is less than 1 and varies from 0.75 to 0.97.

An essentially heteromodular material is concrete [4]. Thus, the coefficient of the heteromodularity of different types of heavy concrete is more than 1 and varies from 1.07 to 1.82. At the same time, the coefficient of heteromodularity of light concrete can be either more or less than 1.

Based on the experimental data obtained f E^{+}/E^{-} or various materials which differently resist tension and pressure, a phenomenological theory [5] was built. It describes the behaviour of a heteromodular material and proposes general methods of solving problems of the theory of elasticity in case of such a material. According to this theory, the diagram of tension-deformation for the materials with different moduli of elasticity, in case of tension and pressure, is represented in the form of a bilinear diagram with a gap in the value of the inclination angle of the tangent in point 0 (Fig. 1).

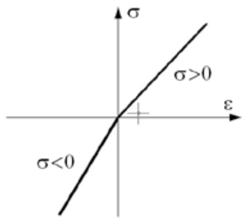

Fig. 1. The diagram of tension-deformation

The heteromodularity of construction material in this case is characterized by the moduli of elasticity ratio

$$\mu = \frac{E^{-}}{E^{+}}.$$

This article deals with the influence of the heteromodularity of material on the parameters of the TDS of a truss with a rigid connection of the rods in joints with a nodal scheme of loading. For the parameters to be determined one should apply a force method and use, as a basic system, a hinge and rod system obtained when notionally replacing rigid joints with hinges. The basic unknown quantities in this case are nodal moments.

The coefficients and free terms forming part of the canonical equations of the force method are calculated according to Maxwell-More's formula which for a heteromodular material, in case of taking into account the flexural and longitudinal deformation, looks like [6]

$$\Delta_i = \sum_k \int_l \frac{m_i M_P}{D} ds + \sum_m \int_l \frac{n_i N_P}{E^- A} ds + \sum_j \int_l \frac{n_i N_P}{E^+ A} ds\ , \tag{1}$$

where D is the flexural rigidity of the cross-section for a heteromodular material or the reduced flexural rigidity.

For the symmetrical cross-section of arbitrary form, the reduced flexural rigidity is determined by the formula

$$D = E^- I^- + E^+ I^+\ , \tag{2}$$

where the quantities I^- and I^+ characterize respectively the moments of inertia in the compressed and stretched parts of the cross-section regarding the neutral axis.

The position of the neutral axis is determined with the help of the equation

$$E^- S^- = E^+ S^+\ , \tag{3}$$

where S^- is the static moment of the compressed part of the cross-section; S^+ is the static moment of the stretched part of the cross-section.

Using the obtained formulas (2), (3), it is possible to determine the position of the neutral line and the reduced flexural rigidity for any of the cross-sections found when making calculations of pivoted constructions. Thus, for example, the position of the neutral line for the rectangular cross-section is described with the correlations

$$h^-(\mu) = \frac{1}{1+\sqrt{\mu}} h;\ \ h^+(\mu) = \frac{\sqrt{\mu}}{1+\sqrt{\mu}} h,$$

where h^-, h^+ are, the height of the compressed and stretched part of the cross-section respectively. Then the reduced rigidity of the rectangular cross-section is described with the a formula of the following kind

$$D = \frac{b}{3}\left(E^-\left(h^-\right)^3 + E^+\left(h^+\right)^3\right).$$

Thus, when making calculations of trusses with the force method, considering the heteromodularity of a material, the formula for calculating the coefficients of canonic equations according to (1) will look like

$$\delta_{ij} = \sum_k \int_l \frac{m_i m_j}{D} ds + \sum_m \int_l \frac{n_i n_j}{E^- A} ds + \sum_j \int_l \frac{n_i n_j}{E^+ A} ds\ , \tag{4}$$

and for calculating the free terms of the equation according to (1), considering the used variant of the basic system of the force method, the formula will look like

$$\Delta_{iP} = \sum_m \int_l \frac{n_i N_P}{E^- A} ds + \sum_j \int_l \frac{n_i N_P}{E^+ A} ds\ . \tag{5}$$

For the numerical evaluation of the influence of the material heteromodularity on the values of the parts of the coefficients of the canonical equations which take into account the influence of the flexural deformations, it is possible to introduce the coefficient of the influence of the material modularity

$$\zeta = \frac{D}{EI_z}\ . \tag{6}$$

In the case of the rectangular cross-section, this coefficient looks like

$$\zeta = \frac{1}{4}\frac{\left(1+\sqrt{\mu}\right)^2}{\mu}$$

In Fig. 2 the graph shows the dependence of this coefficient on the quantity μ

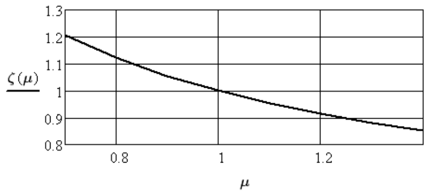

Fig. 2. The dependence of the influence coefficient on μ

Let's give the numerical evaluation of the influence of heteromodularity on the parameters of the TDS of a truss with rigid joints, using a particular example. Let's consider a two-pivoted truss with rigid connection in the joint under the influence of arbitrary nodal loading. (Fig. 3)

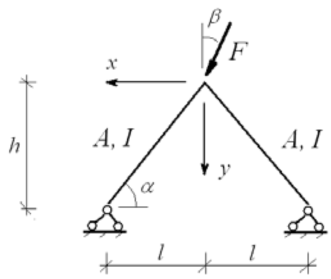

Fig. 3. Two-pivoted truss

The calculation of the truss is carried out with the force method. In the capacity of a basic system one should take a truss with a hinged connection of the rods in the joint (Fig. 4)

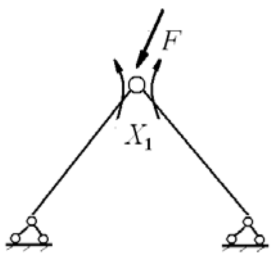

Fig. 4. Basic system of the force method

The canonical equation of the force method looks like

$$\delta_{11}X_1 + \Delta_{1P} = 0$$

The unknown quantity in the equation (1) is the nodal bending moment.

The inner efforts of the solitary state, which are connected with the calculation of the coefficient and free term of the equation, in the dimensionless form, look like this:

– the bending moments

$$m_1(x,l) = (1 - \frac{x}{l});\ m_2(x,l) = (1 - \frac{x}{l})$$

– the longitudinal forces

$$n_1'(\alpha) = \cos\alpha;\ n_2'(\alpha) = \cos\alpha$$

The longitudinal forces of the loading state, which are connected with the calculation of the free term, in the dimensionless form, look like this

$$N_1'(\alpha,\beta) = -0.5\frac{\cos\beta}{\sin\alpha}(1+\frac{tg\beta}{tg\alpha});\ N_2'(\alpha,\beta) = 0.5\frac{\cos\beta}{\sin\alpha}(\frac{tg\beta}{tg\alpha}-1),$$

and the bending moments in the loading state, don't appear.

If we take into account the calculation of the coefficient by the formula (4) and the free term by the formula (5) the nodal bending moment looks like:

- in case $\beta \le \alpha$

$$X_{1\mu}'(\alpha,\beta,\lambda,\mu) = 1.5\frac{\zeta\cos\beta}{\cos^2\alpha(3\zeta+\lambda^2\mu tg^2\alpha)}$$

- in case $\beta \ge \alpha$

$$X_{1\mu}'(\alpha,\beta,\lambda,\mu) = 0.75\left[(1+\mu)+(1-\mu)\frac{tg\beta}{tg\alpha}\right]\frac{\zeta\cos\beta}{\cos^2\alpha(3\zeta+\lambda^2\mu tg^2\alpha)}$$

The dependence of the quantity of the obtained nodal bending moment on the parameters of the truss α, β, λ, μ is represented in the graphs in Fig. 5

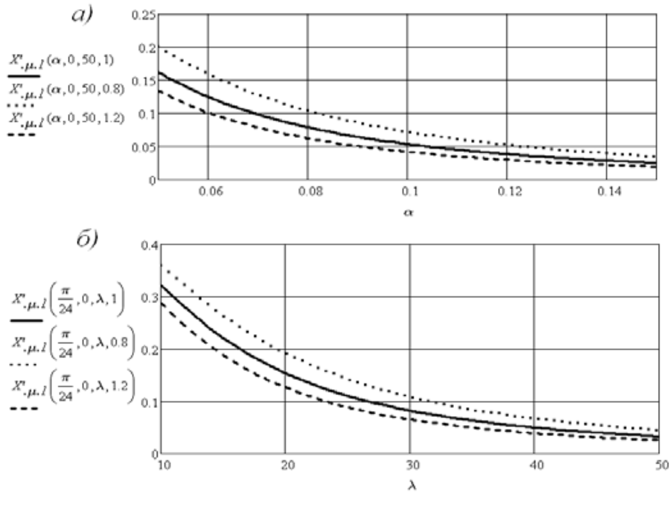

Fig. 5. The graphs showing the dependence of the quantity of the nodal bending moment on the parameters of the truss

It follows from the graphs in Fig. 5.a that the quantity of the nodal bending moment increases with the decrease of angle α and decreases with the increase of angle β that is with the appearance of the horizontal component of the nodal loading in the joint. From the graphs in Fig. 5.b it follows that the quantity of the nodal bending moment increases with the decrease of the flexibility in the truss rods. Besides, the quantity of the nodal bending moment in the truss essentially depends on the coefficient of the heteromodularity of the material μ. At values $\mu < 1$ the nodal moment increases, and at values $\mu > 1$ it decreases in comparison with the quantity of this moment disregarding the heteromodularity of the material.

The dimensionless increases in the longitudinal forces in the truss rods, which appear as a result of the nodal bending moment, for a heteromodular material described with the formulas

$$\Delta N_{1\mu}(\alpha,\beta,\lambda,\mu) = 1 - k_{1\mu}(\alpha,\beta,\lambda,\mu);$$
$$\Delta N_{2\mu}(\alpha,\beta,\lambda,\mu) = 1 - k_{2\mu}(\alpha,\beta,\lambda,\mu),$$

where

$$k_{1\mu}(\alpha,\beta,\lambda,\mu) = 1 + \frac{n_1' X_{1\mu}'(\alpha,\beta,\lambda,\mu)}{N_{1P}'};$$
$$k_{2\mu}(\alpha,\beta,\lambda,\mu) = 1 + \frac{n_2' X_{1\mu}'(\alpha,\beta,\lambda,\mu)}{N_{2P}'}$$

are the coefficients of the influence of the rigidity of the joint on longitudinal forces in the truss rods, the heteromodularity of the material taken into account.

The dependence of the increases of the longitudinal forces on the parameters of the truss α,β,λ,μ is presented in the graphs in Fig. 6

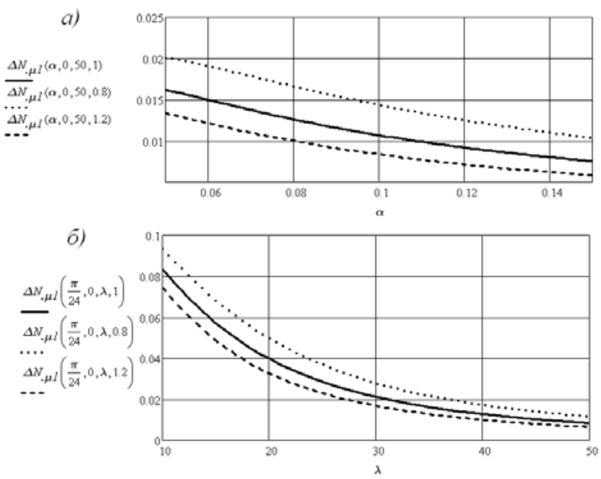

Fig. 6. The graphs showing the dependence of the quantity
of the longitudinal forces increase on the parameters of the truss.

From the above graphs it follows that in case of the most unfavourable combination of the parameters of the truss, the quantity of the increase of the longitudinal forces doesn't exceed 10% of the values of the longitudinal forces according to the hinged scheme of the truss. However, at the values $\mu < 1$ this increase grows and at the values $\mu > 1$ it falls in comparison with the quantity of the increase of the longitudinal forces disregarding the heteromodularity of the material.

The dimensionless increases of the normal tensions in the truss rods which appear as a result of the nodal bending moment, for the heteromodular material are described with the formulas

$$\Delta\sigma_{1\mu}(\alpha,\beta,\lambda,\mu) = 1 - k_{1\sigma\mu}(\alpha,\beta,\lambda,\mu);$$
$$\Delta\sigma_{2\mu}(\alpha,\beta,\lambda,\mu) = 1 - k_{2\sigma\mu}(\alpha,\beta,\lambda,\mu),$$

where

$$k_{1\sigma\mu}(\alpha,\beta,\lambda,\mu) = k_{1\mu}(\alpha,\beta,\lambda,\mu) + \lambda\cos\alpha\,\frac{X_{1\mu}'(\alpha,\beta,\lambda,\mu)}{N_{1P}'(\alpha,\beta)};$$
$$k_{2\sigma\mu}(\alpha,\beta,\lambda,\mu) = k_{2\mu}(\alpha,\beta,\lambda,\mu) + \lambda\cos\alpha\,\frac{X_{1\mu}'(\alpha,\beta,\lambda,\mu)}{N_{2P}'(\alpha,\beta)}$$

are the coefficient of the influence of the rigidity of the joint on the normal tensions in the truss rods. The dependence of the increases of the normal tensions on the parameters of the truss α, β, λ, μ is presented in the graphs in Fig. 7

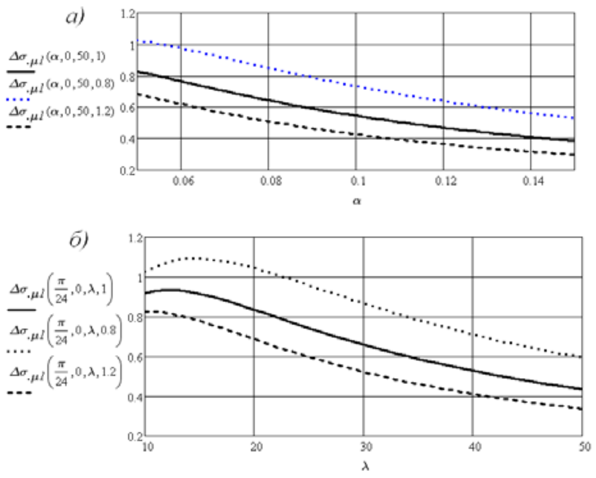

Fig. 7. The graphs showing the dependence of the increase of the normal tensions on the parameters of the truss

From the presented graphs it is possible to draw the following conclusions. Firstly, when taken into account the heteromodularity of the construction material of the truss rods may have a noticeable influence on the growth of the increases of the normal tensions as a result of the appearance of the nodal bending moment. Secondly, a significant influence on the manifestation of the heteromodularity effect is caused by the parameters of the calculation scheme of the truss.

REFERENCES

1. Земляков, И.П. О различии модулей упругости полиамидов при различных видах деформации / И.П. Земляков. – Механика полимеров. – 1965. – № 4.
2. Гольдман, А.Я. Исследование механических свойств тканевых стеклопластиков при растяжении и сжатии нормально к плоскости армирования / А.Я. Гольдман, Н.Ф. Савельева, В.И. Смирнова. – Механика полимеров. – 1968. – № 5.
3. Иванов, Г.П. Исследование несовершенной упругости металлов : автореф. дис. / Г.П. Иванов. – Минск, 1973.
4. Авхимков, А.П. Об уравнениях обобщенного закона упругости материалов, разносопротивляющихся растяжению и сжатию и некоторых их приложениях : автореф. дис. / А.П. Авхимков. – М., 1975.
5. Амбарцумян, С.А. Разномодульная теория упругости / С.А. Амбарцумян. – М. : Наука, 1982.
6. Турищев, Л.С. К расчету стержневых конструкций с учетом влияния разномодульности материала / Л.С. Турищев // Вестник Полоцкого государственного университета. Сер. В. Промышленность. Прикладные науки. – 2011. – № 11.