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MATHEMATICAL MODELLING OF PROCESSES OF COMBINED STRESS TAKING INTO ACCOUNT CHANGES OF ELASTIC PROPERTIES AT PLASTIC DEFORMATION

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The change of elastic constants of material owing to elastic plastic deformation is considered in the article. Therefore, the elastic and plastic components of full deformation change and define borders of an elastic kernel. An attempt to determine approaches using changing elasticity modules at mathematical modeling of processes of combined stress is made, expressions of modules of elasticity E are given at a clean bend, G shift – at a cross bend and torsion on condition of transition of material from an elastic state to plastic.

At mathematical modeling of process of combined stress of metal designs and conditions of plasticity the settlement resistance to the bearing ability of $R_d = R_K / \sigma_M$, which is established on the basis of tests of samples at monoaxial loading is manifested in the inequality of a technique of limit states.

However, material in real designs, as a rule, is in a combined stress. In this connection, it is necessary to establish the rule of equivalence of combined stress to monoaxial. As an equivalence criterion, we use the expression of potential energy which accumulates in the material when it is deformed by external influences. The deformation energy we present is the sum of products of volume of A_0 and change of a shape of body A_f in the form.

$$A_i = A_0 + A_f \tag{1}$$

$$A_0 = \frac{1-2v}{6E} \cdot (\sigma_X + \sigma_Y + \sigma_Z)^2, \tag{2}$$

$$A_f = \frac{1+v}{3E} \cdot \left[\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2 - (\sigma_X\sigma_y + \sigma_Y\sigma_Z + \sigma_Z\sigma_X) + 3({\tau_{xy}}^2 + {\tau_{yz}}^2 + {\tau_{zx}}^2) \right]. \tag{3}$$

Work at monoaxial tension:

$$A_e = \frac{1+v}{3E} \cdot \sigma^2. \tag{4}$$

The ratio of power equivalence of combined stress to monoaxial is:

$$\sigma = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2 - (\sigma_X\sigma_Y + \sigma_Y\sigma_Z + \sigma_Z\sigma_X) + 3({\tau_{xy}}^2 + {\tau_{yz}}^2 + {\tau_{zx}}^2)} \tag{5}$$

Then the generalized coefficient E_{ob} at combined stress will be defined:

$$E_{ob} = \frac{\sigma}{\varepsilon_{ob}} \tag{6}$$

Formulas for definition of E and G on condition of transition of material from an elastic state in plastic are given below:

Central stretching:

$$E = \frac{1}{\varepsilon} \cdot \sum \int_0^1 \frac{\overline{N_l} \cdot N_p}{F} \cdot dS \tag{7}$$

Central compression:

$$E = \frac{\sigma_{cy} \cdot {l_o}^2}{n^2 \cdot \pi^2 \cdot i^2} = \frac{\sigma_{cy} \cdot \lambda^2}{n^2 \cdot \pi^2}; \lambda \geq 100. \tag{8}$$

$$T = \frac{\sigma_{cy} \cdot \lambda_{pr}^{\ 2}}{n \cdot \pi^2}; \lambda \prec 100. \tag{9}$$

where T – the given deformation module; λ_{pr} – the given flexibility.

At a clean bend:

$$E = \frac{1}{\Delta_{ip}} \cdot \sum \int_0^1 \frac{\overline{M_l} \cdot M_p}{I} \cdot dS \tag{10}$$

At shift:

$$G = \frac{1}{\Delta_{ip}} \cdot \sum \int_0^1 \frac{\overline{Q_l} \cdot Q_p}{F} \cdot dS$$

Here:

$$\Delta_{ip} = \frac{1}{\sin y} \cdot \sqrt{\Delta_{1p}^{\ 2} - 2\Delta_{1p} \cdot \Delta_{2p} \cdot \cos y + \Delta_{2p}^{\ 2}} \tag{11}$$

γ-corner between movements Δ_{1p} and Δ_{2p}

The change of elastic properties has been noted in a number of experimental works where the decrease of modules of elasticity of E and G in the course of elastic plastic deformation is stated. At the same time, on the way of unloading and the subsequent loading along with elastic and plastic deformations, the hysteresis loop appears. It is necessary to achieve disappearance of a loop by cyclic loading - unloading, and consider deformations registered after that when determining the module of elasticity. According to the offered technique, samples were deformed up to various sizes, at various range and scope of tension. The results of the modules change were almost identical. To determine the influence of size of preliminary plastic deformation plastic deformation 0,014 was reported to a sample. The change of the module G has made at the same time 6,2%. The sample was in addition deformed to 0,0243. Reduction of the module has made another 3,2%, i.e. the general reduction of the module of 9,4%, the obvious correlation between the size of initial deformation and size of change of the module of elasticity is observed. Thus, the decrease in the modules of elasticity depends only on the size of initial plastic deformation.

This conclusion allows us to use the changed elasticity modules at mathematical modeling, obtained experimentally, which allows more accurate consideration of the properties of the material.

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