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ABOUT THE DIFFERENTIAL EQUATION OF BENDING COMPRESSED-BENT BAR TAKING INTO ACCOUNT THE DIFFERENT MODULAS OF MATERIAL

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The article shows the synthesis of the differential equation of axis of the compressed-bent bar for a case when engineering material of a beam is characterized by various modulus of elasticity when stretch and compression. Using the method of initial parameters generalized formulas were received to determine parameters of the stress-strain state in an arbitrary section of bar.

Consider a rectilinear flexible bar which is loaded with a constant longitudinal force and transverse load, shown in Figure 1 below.

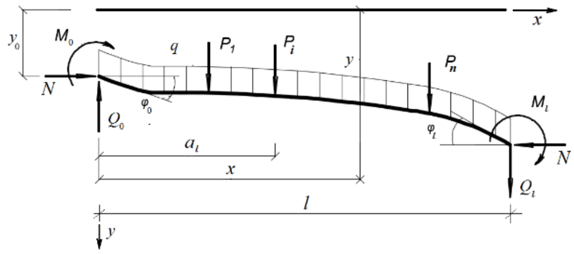

Fig. 1. The compressed-bent bar

The bar undergoes deformations of compression with bending under the action of the applied load. The stress-strain state of the bar at the origin is characterized by the quantities

$$y(0)=y_0\,,\ y'(0)=y'_0\,,\ M(0)=M_0\,,\ Q(0)=Q_0$$

which are, respectively, the deflection, the turning angle, the bending moment and the shear in the left section of bar, and are called the initial parameters.

The adopted that the bar is made of a differentmodulas material. The section of the bar is symmetrical of arbitrary shape (Fig. 2)

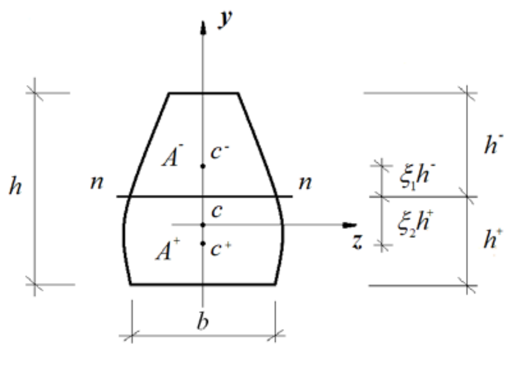

Fig. 2. Symmetrical transverse section of bar

where A^- – the area of the compressed part of the section with the center of gravity at the point c^-; A^+ – the area of the stretched part of the section with the center of gravity at the point c^+. The total area of section $A=A^-+A^+$.

To describe the deflections of the compressed-bent bar made from a differentmodulas material, we use the approximate differential equation of bent axis of such a bar [1]

$$\frac{d^2y}{dx^2}=-\frac{M(x)}{D(x)}, \tag{1}$$

where $M(x)$ – the bending moment in an arbitrary section; $D(x)$ – the bending stiffness of the transverse section of the bar, taking into account the differentmodulas of material.

The bending moment $M(x)$ entering into equation (1), for the considered load distribution of the bar is described by expression

$$M(x)=M_0+Q_0x+N[y(x)-y_0]-\sum P_i(x-a_i)-\frac{qx^2}{2} \tag{2}$$

In determining the moment from formula (2) in some section, only those forces that are to the left of this section must be taken into account under the summation sign.

The quantity that enters into equation (1) $D(x)$, characterizes the bending stiffness of the transverse section, taking into account the influence of the differentmodulas of the engineering material. This value is related to the bending stiffness of the transverse section EI_z by the ratio [2]

$$D=\frac{EI_z}{\zeta} \tag{3}$$

where ζ – influence coefficient of the differentmodulas of the engineering material on the bending stiffness of the transverse section. For a symmetric transverse section of an arbitrary shape, it is described by expression

$$\zeta=\frac{I_z}{I^+(x)+\mu I^-(x)}$$

where $I^-(x)$ and $I^+(x)$ characterized by moment of inertia, respectively, compressed and stretched parts of the transverse section with respect to the neutral axis; $\mu=\frac{E^-}{E^+}$ – coefficient of differentmodulas of engineering material, characterizing the ratio of its modulus of elasticity during compression E^- and stretching E^+.

Taking (2), (3) into account, the equation (1) takes the next form

$$\frac{d^2y}{dx^2}+\zeta\frac{N}{EI_z}y=-\frac{\zeta}{EI_z}\left[M_0+Q_0x-Ny_0-\sum P_i(x-a_i)-\frac{qx^2}{2}\right] \tag{4}$$

The resulting equation (4) describes the of the bent axis of the flexible compressed-bent bar, taking into account the differentmodulas of its material and is an ordinary inhomogeneous differential equation of order 2 with constant coefficients. With the value of $\zeta=1$ equation (4) takes the form when the calculation of the compressed-bent bar is conducted without taking into account the differentmodulas of material.

Introducing the notation $n=\sqrt{\frac{N}{EI_z}}$, we obtain the general solution of equation (4) in the form

$$y(x)=C_1\cos\sqrt{\zeta}nx+C_2\sin\sqrt{\zeta}nx+y^*(x). \tag{5}$$

where C_1, C_2 – the arbitrary constants; $\cos\sqrt{\zeta}nx$, $\sin\sqrt{\zeta}nx$ -the partial solutions of the homogeneous equation obtained from equation (4) $y^*(x)$ - the particular solution of the inhomogeneous equation (4), which has the form

$$y^*(x)=y_0-\frac{M_0+Q_0x}{n^2EI_z}+\frac{\sum_i P_i}{n^3EI_z}\left[\sqrt{\zeta}n(x-a_i)-\sin\sqrt{\zeta}n(x-a_i)\right]-\frac{q}{n^4EI_z}\left[\left(1-\frac{\zeta n^2x^2}{2}\right)-\cos\sqrt{\zeta}nx\right]$$

The arbitrary C_1, C_2 are determined from the boundary conditions at the left end of the bar

$$y(0)=y_0$$
$$y'(0)=y'_0$$

Using these conditions, we find

$$C_1 = \frac{M_0}{n^2 EI_z}, \; C_2 = \frac{1}{\sqrt{\zeta} n}\left(\varphi_0 + \frac{Q_0}{n^2 EI_z}\right). \tag{6}$$

Substituting (6) into the general solution (5) we obtain the formula for deflections of a compressed-bent bar

$$y(x) = y_0 + y_0' \frac{\sin\sqrt{\zeta}nx}{\sqrt{\zeta}n} + \frac{M_0}{n^2 EI_z}\left(\cos\sqrt{\zeta}nx - 1\right) + \frac{Q_0}{n^3 EI_z}\left(\frac{\sin\sqrt{\zeta}nx}{\sqrt{\zeta}} - nx\right) + y_P(x), \tag{7}$$

Differentiating successively (7), we obtain formulas for the turning angle, the bending moments and the transverse forces of the compressed-bent bar

$$\begin{aligned} y'(x) &= y_0' \cos\sqrt{\zeta}nx - \frac{M_0}{nEI_z}\sqrt{\zeta}\sin\sqrt{\zeta}nx + \frac{Q_0}{n^2 EI_z}\left(\cos\sqrt{\zeta}nx - 1\right) + y_P'(x) \\ M(x) &= y_0' \frac{EI_z}{\sqrt{\zeta}} n \sin\sqrt{\zeta}nx + M_0 \cos\sqrt{\zeta}nx + Q_0 \frac{\sin\sqrt{\zeta}nx}{n\zeta} + M_P(x) \\ Q(x) &= y_0' EI_z n^2 \cos\sqrt{\zeta}nx - M_0\sqrt{\zeta}n \sin\sqrt{\zeta}nx + Q_0 \frac{\cos\sqrt{\zeta}nx}{\sqrt{\zeta}} + Q_P(x) \end{aligned} \tag{8}$$

In formulas (7) and (8), which allow one to find the parameters of the stress-strain state in an arbitrary section of a compressed-bent bar, taking into account the influence of the differentmodulas of engineering material the quantities $y_P(x)$, $y_P'(x)$, $M_P(x)$, $Q_P(x)$ characterize the influence of the load in the span on the corresponding parameter of the stress-strain state and for the load shown in Fig. 1 are determined by the following formulas:

$$\begin{aligned} y_P(x) &= \frac{1}{n^3 EI_z}\sum_i P_i\left[\sqrt{\zeta}n(x - a_i) - \sin\sqrt{\zeta}n(x - a_i)\right] - \frac{q}{n^4 EI_z}\left[\left(1 - \frac{\zeta n^2 x^2}{2}\right) - \cos\sqrt{\zeta}nx\right] \\ y_P'(x) &= \frac{\sqrt{\zeta}}{n^2 EI_z}\sum_i P_i\left[1 - \cos\sqrt{\zeta}n(x - a_i)\right] + \frac{q}{n^3 EI_z}\left[\zeta nx - \sin\sqrt{\zeta}nx\right] \\ M_P(x) &= -\frac{1}{n}\sum_i P_i \sin\sqrt{\zeta}n(x - a_i) - \frac{q}{\zeta n^2}\left[\zeta - \sqrt{\zeta}\cos\sqrt{\zeta}nx\right] \\ Q_P(x) &= -\zeta\sum_i P_i \cos\sqrt{\zeta}n(x - a_i) - \frac{q}{n}\sin\sqrt{\zeta}nx \end{aligned}$$

The obtained differential equation (4) and formulas (7), (8) allow to obtain solutions to specific tasks of computation of compressed-bent bar, taking into account the differentmodulas of material.

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