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DIFFERENTIAL EQUATION OF HEAT CONDUCTIVITY AND ITS SOLUTION IN METALWORKING

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Introduction. In connection with constant updating types of structural materials, increasing of requirements to quality of machine parts and intensification processing modes the issues related to thermal processes in technological systems come to the fore.

To consider these processes when designing the process equipment, to be able to manage them, you need to know the heat transfer in the elements of the technological system.

Principal of heat transfer. There are three types of heat transfer: conduction, convection and thermal radiation. [1]

Conduction is the process of propagation of heat energy by direct contact of bodies or separate body parts, with different temperatures, for example, workpieces, tools and parts manufacturing equipment.

Convection is the process of moving volumes of liquid or gas in the space from one area to another, with different temperature.

Convection of heat is always accompanied by conduction, as the movement of liquid or gas means contacts of the particles having different temperatures.

A collaborative process of convection and conduction is called heat transfer.

Thermal radiation is the process of heat propagation in the form of electromagnetic waves with the mutual conversion of thermal energy to radiant and back again.

An example of the types of heat transfer during cutting is presented in figure 1, where C - conductivity, R - radiation, convection, CWF – convection with the fluid, CWA -convection with the air; ON, OL and OS – lots of heat from the deformation, the friction in the front and rear surfaces of the cutting tool.

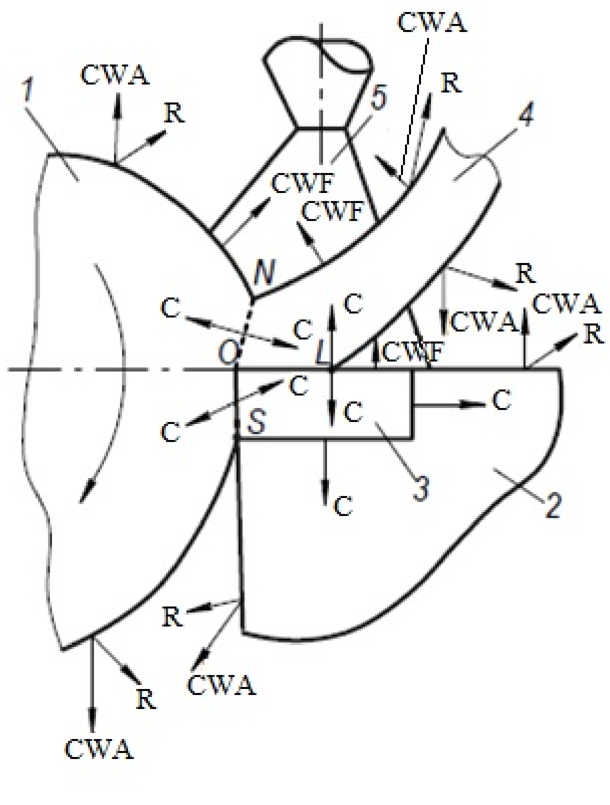

Fig. 1. Types of heat exchange in the cutting zone:
1 – workpiece; 2 – cutting tool; 3 – cutting insert; 4 – shavings; 5 – cutting fluid

The quantity of heat dQ flowing through isothermal surface with an area of dF for the time dτ is proportional to the temperature gradient (figure 2):

$$dQ = -\lambda grad\theta F \cdot d\tau. \qquad (1)$$

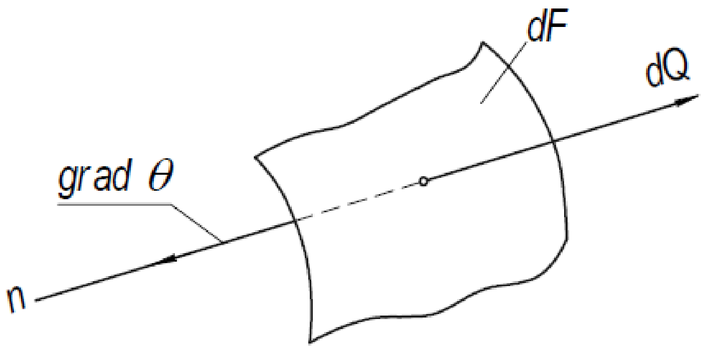

Fig. 2. The insulated element and surface heat flux

The amount of heat passing through unit area of the isothermal surface in unit time (heat flux density), is determined by the ratio:

$$q = \frac{dQ}{dF \cdot d\tau} = -\lambda grad\theta. \qquad (2)$$

where λ - the coefficient of heat conductivity expresses the quantity of heat passing in unit time through unit area when the temperature gradient of one degree per unit length.

The thermal conductivity characterizes the ability of the material to conduct heat and depends on the composition of the substance, its structure, density, humidity and temperature. The minus sign in equation (2) shows that the vector of the heat flow directed in the direction opposite to the direction of the vector gradθ. Expression (2) typically represents the basic law of heat conduction, or Fourier law stating that the heat flux density is directly proportional to the temperature gradient.

The differential equation of heat conduction. In unsteady mode the redistribution of heat is accompanied by a change in temperature of the separate elements of the body. The change in the temperature fields of solid bodies at non-stationary heat conduction is described by the differential equation. For the derivation of this equation in the body of the elementary volume ΔxΔyΔz (figure. 1) 100, the process of propagation of heat in it is considered in the direction of one axis, for example OX.

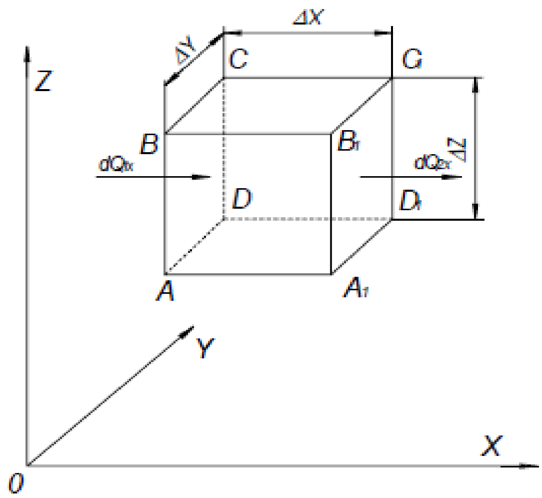

Fig. 3. Scheme to the conclusion of the differential equation of heat conductivity

The smallness of area ΔyΔz, supposes that the temperature on each face of the body is distributed evenly. Let face A1B1C1D1 temperature is θ, and on the verge $ABCD - \theta' = \theta + \Delta\theta_X, \Delta\theta_X$, where the temperature change along the OX axis. The change of the heat flow from one face to another direction OX consider the body described by the expression:

$$dQ_{2x} = \lambda \text{grad}_x \theta \cdot \Delta y \Delta z \Delta d\tau$$

or

$$dQ_{2x} = \lambda \frac{\partial \theta}{\partial x},$$

The temperature of the faces ABCD:

$$\theta' = \theta + \Delta x \frac{\partial \theta}{\partial x'},$$

then

$$\text{grad}_x \theta' = \frac{\partial \theta'}{\partial x} = \frac{\partial}{\partial x}\left(\theta + \Delta x \frac{\partial \theta}{\partial x}\right)$$

and heat flow

$$dQ_{1x} = \lambda \frac{\partial}{\partial x}(\theta + \Delta x \frac{\partial \theta}{\partial x}) \Delta y \Delta z d\tau.$$

as dQ1x > dQ2x , in the amount of

$$dQ_x = dQ_{1x} - dQ_{2x} = \lambda \frac{\partial \theta}{\partial x} \Delta y \Delta z d\tau + \lambda \frac{\partial^2 \theta}{\partial x^2} \Delta y \Delta z \Delta x d\tau - \lambda \frac{\partial \theta}{\partial x} \Delta y \Delta z d\tau = \lambda \frac{\partial^2 \theta}{\partial x^2} \Delta x \Delta y \Delta z d\tau$$

or

$$dQ_x = \lambda \frac{\partial^2 \theta}{\partial x^2} = dV d\tau, \quad (3)$$

Similarly, for the coordinate axes OY and OX:

$$dQ_y = \lambda \frac{\partial^2 \theta}{\partial y^2} = dV d\tau, \quad (4)$$

$$dQ_z = \lambda \frac{\partial^2 \theta}{\partial z^2} = dV d\tau, \quad (5)$$

The change of the heat flow in the whole body will be described by the expression:

$$dQ = dQ_x + dQ_y + dQ_z = \lambda(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}) dV d\tau, \quad (6)$$

The amount of heat can also be determined using the heat capacity:

$$dQ = \rho \cdot c \cdot dV(\frac{\partial \theta}{\partial \tau}) d\tau. \quad (7)$$

where ρ – the density of the material;
c – mass heat capacity;
ρ·c - volumetric heat capacity;
$\frac{\partial \theta}{\partial \tau}$ – the rate of change of temperature of the body in time.

Equating the expression (6) and (7), we obtain the differential heat conduction equation of Fourier:

$$\frac{\partial \theta}{\partial \tau} = \omega(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}). \quad (8)$$

where $\omega = \frac{\lambda}{c \cdot \rho}$ – the thermal diffusivity characterizing the thermal inertia properties of the body.

This equation is general and valid for any body shape and any conditions of heat transfer.

Due to the given above assumptions, the conduction equation can be solved in one of the following methods: classical, operating, numerical, heat sources and modeling.

When using the classical method, the integration of the differential equation (8) is one of the well-known mathematical methods. Operating methods, for example, in the method of integral Laplace transform do not study the function we are interested in, and its modification is obtained by multiplying on an exponential function. When using numerical methods for differential heat conduction equation is solved by the finite difference method. It is based on replacing the derivatives in equation (8) by their approximate values, expressed by differences of the functions θ(x, y, z,τ) in discrete points-nodes of the grid (after the body is split into elementary volumes of the correct form). The differential equation in this case is replaced by the equivalent ratio of finite differences, for the solution of which the algebraic operations are implemented on the computer. To determine the temperature at any point of the body in a given time is enough to know the temperature of the neighboring points at the previous time. If you know the initial temperature distribution in the body and on its boundary surfaces, then gradually, step by step, moving from one point of the body to another, or considering one time after another, the temperature field in the object can be calculated.

The most widely used method in various fields of engineering is the method for the heat sources. The essence of this method is determined by two main provisions:

1) temperature field arising in a heat-conducting body under the influence of a heat source of any shape, moving or stationary, when the current is alternating or continuous, can be obtained as the result of a particular combination of temperature field arising under the action of instantaneous point sources;

2) the process of propagation of heat in the body is in the form of the distribution of heat in the body of unlimited size by adding to the actually existing sources a system of fictitious sources or heat sinks.

If we assume that in the body, all points of which have the same temperature, and the heat exchange with the environment misses, with the broken or instantly extinguished point source contributing the qT of heat, the solution of the differential equation (8) for these conditions can be written as:

$$\theta_T(x,y,z,\tau) = -\frac{q_T}{\lambda\sqrt{\omega}(4\pi\tau)^{3/2}} \exp - \frac{-(x_u-x)^2+(y_u-y)^2+(z_u-z)^2}{4\omega\tau}. \quad (9)$$

where $\theta_T(x,y,z,\tau)$ – the temperature of any point of the body;

x, y, z – the coordinates of the point of the body;

$x_u y_u z_u$ – the coordinates of the heat source;

τ – the duration of the source;

λ and ω – the coefficients of thermal conductivity and thermal diffusivity of the body material.

Modeling of thermal processes are carried out mainly in two types:

1) physical modeling when studying the heat transfer process in a real body is based on analysis of similar process of propagation of heat in the model;

2) mathematical modeling when studying heat transfer in a real body is based on the analysis of fundamentally different physical phenomena, different from the process of propagation of heat, but having a similar mathematical description.

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