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METHODS AND ALGORITHMS FOR EVALUATING THE RELIABILITY OF BUILDING STRUCTURES BASED ON PROBABILISTIC MODELS OF DYNAMIC LOADS

KANSTANTSIN APANCHONAK
Polotsk State University, Belarus

In this paper, we consider the basic models and types of dynamic loads on building structures, as well as the types of distributions for generators of random variables used to construct these models.

Modern approaches to assessing the reliability of buildings and structures in their design, survey, design of amplification and reconstruction recommend the use of new calculation methods based on probabilistic models. Today we observe the lack of knowledge about the variability of certain parameters of building structures, loads, and indirect influence of environmental parameters. At this stage of building science development, probabilistic models can serve as tools for simulation modeling in the calibration of the coefficients of semi-probability methods of calculation.

In the construction of models different random number generators are used to obtain the most approximate values of existing systems. Software generators of uniformly distributed numbers calculate each new number based on one or more preceding numbers in accordance with a given mathematical formula. Thus, the resulting numbers are completely deterministic and it is possible to repeat the run with the same sequence of values obtained.

Random number generation can be done for different kinds of distributions.

UNIFORM DISTRIBUTION

A uniform distribution is a continuous random variable that has a uniform distribution law if its density fX (x) has the form:

$$fX(x) = \begin{cases} \frac{1}{b-a}, & x \in [a,b] \\ 0, & x \notin [a,b] \end{cases}$$

A uniform distribution can be used for generating almost any random variable. Figures 1 and 2 show the graphical characteristics of this distribution.

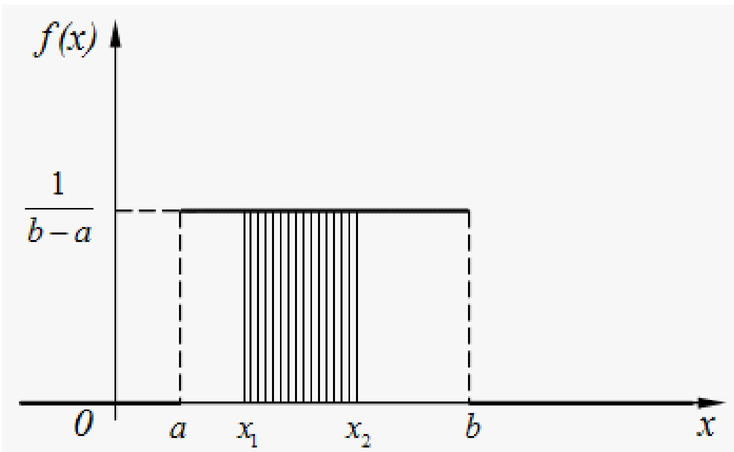

Fig. 1. Graphical characteristics of a uniform distribution of random variables

In the case of a simple random choice, it is assumed that each number is extracted from the general population, uniformly distributed in the interval from 0 to 1.

The mathematical expectation of a uniform distribution:

$$\mu = \frac{a+b}{2}$$

The dispersion of a uniform distribution:

$$\sigma^2 = (b-a)^2/12$$

The standard deviation of a uniform distribution:

$$\sigma = \sqrt{\frac{(b-a)^2}{12}}$$

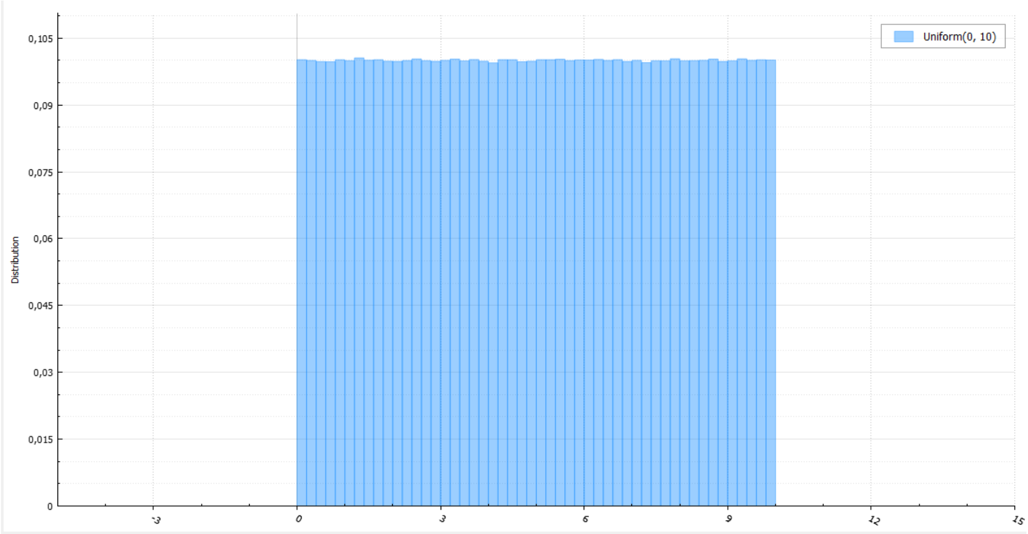

Fig. 2. Graphical characteristics of a uniform distribution of random variables

NORMAL DISTRIBUTION

A normal distribution (or Gaussian / Gauss-Laplace distribution) is the probability distribution, which in the simultaneous case is given by a probability density function that coincides with the Gaussian function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Figure 3 shows the graphical characteristics of this distribution.

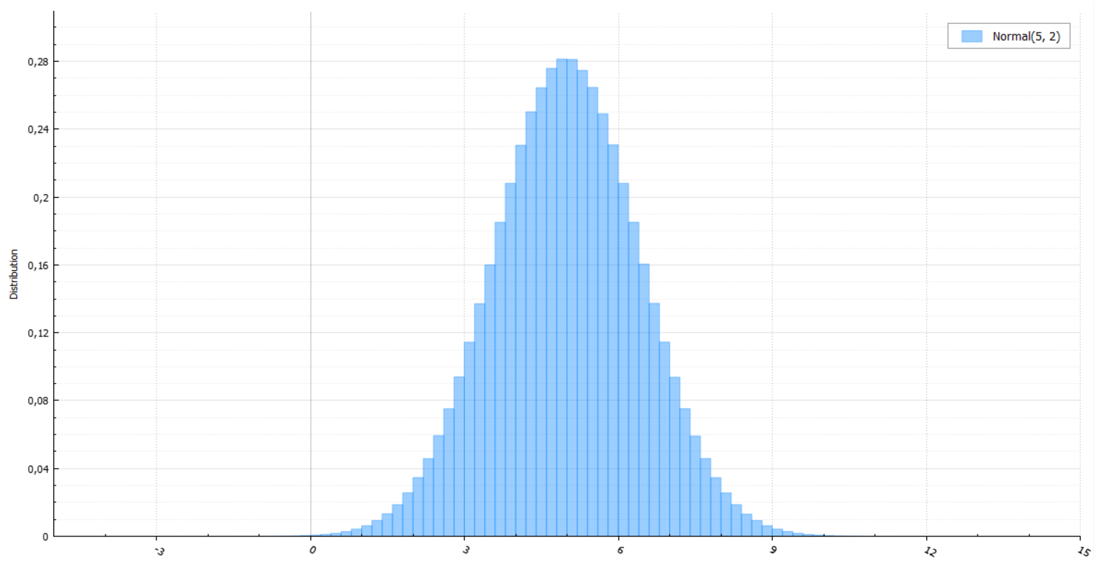

Fig. 3. Graphical characteristics of a normal distribution of random variables

The normalized randomly distributed random variable is obtained from the formula:

$$Z = (X - \mu)/\sigma$$

The expectation of a normal distribution is zero, and the standard deviation is unity. The density of a normal distribution can be obtained by substituting the formula of a random variable into the density formula:

$$f(X) = \frac{1}{\sqrt{2\pi\sigma}}^{\frac{1}{2}z^2}$$

This distribution is used in many fields of science, for example in mathematical statistics and statistical physics. If the result of observation is the sum of many random weakly interdependent quantities, each of which makes a small contribution with respect to the total sum, then as the number of summands increases, the distribution of the centered and normalized result tends to normal.

A normal distribution is often found in nature, for example: deflection in shooting, measurement errors and certain characteristics of living organisms.

WEIBULL DISTRIBUTION

The Weibull distribution is a two-parameter family of continuous distributions having the parameter σ - the shape parameter and λ – the scale parameter. The probability density of a given distribution has the form:

$$fX(x) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The graphical characteristics of this distribution is shown in Figure 4.

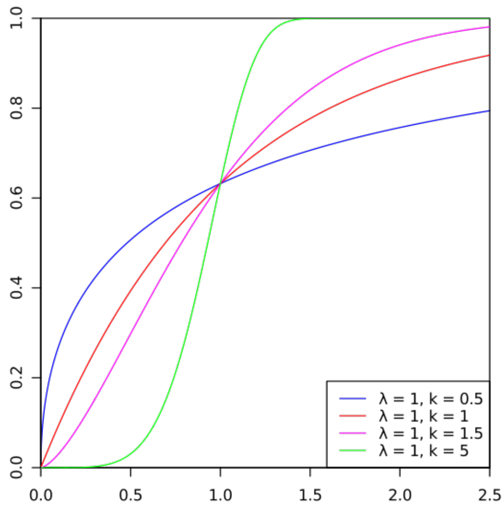

Fig. 4. Graphical characteristicsw of Weibull distribution

This distribution is used in: survival analysis, pullback reliability and analysis, in electrical engineering for representing overvoltage in electrical circuits, in weather forecasting, in predicting technological changes.

DISTRIBUTION OF GUMBEL

The Gumbel distribution (the double exponential distribution) is the probability distribution of a continuous random variable X with a distribution function:

$$f(x) = \exp(-e^{-y}),$$

where $-y < x < y$, $\quad y = (x - a)/b$ and the parameters $-y < a < y,\ b > 0.$

The distribution functions and the graphical representation of the realization of random numbers with the Gumbel distribution are shown in Figures 5 and 6.

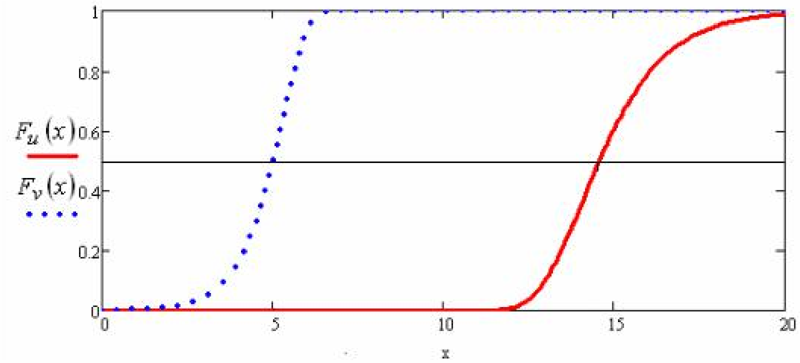

Fig. 5. Gumbel distribution functions: 1 – maximum values; 2 – minimum values

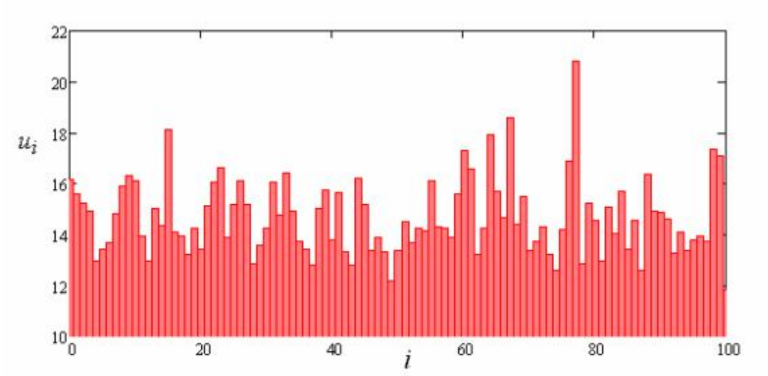

Fig. 6. Graphical representation of the realization of random numbers with the Gumbel distribution

This distribution is used mainly in the statistical analysis of snow and wind loads on structures.

LOGARIFICALLY NORMAL DISTRIBUTION

This distribution is a two-parameter family of absolutely continuous distributions. If the random variable has a lognormal distribution, then its logarithm has a normal distribution. The density of a given distribution has the form:

$$f_X(x) = \begin{cases} \frac{1}{x\sigma\sqrt{2\pi}} e^{-(\ln x - \mu)^2/2\sigma^2}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

where $\sigma > 0, \mu \in R$. Then we say that X has a lognormal distribution with parameters μ and σ.

The density and distribution functions with the parameters μ = 0, σ = 0.7 are shown in Fig. 7.

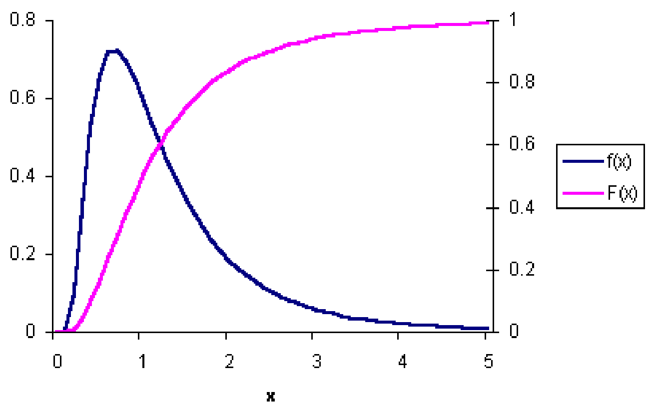

Fig. 7. Graph of the density and functions of the lognormal distribution

Lognormal distribution is used, for example, in the modeling of variables such as income, permissible deviation from the standard of harmful substances in food and the like.

To construct probabilistic models of dynamic loads, examples are given for Gumbel, Weibull, and Lognormal distributions.

Load plays a major role in calculating the reliability of building structures. Loads are of several types: permanent and dynamic. Depending on the type of load, different models are built for calculation.

Constant loads are loads that do not depend on external factors. These loads include the weight of the structures, stationary equipment, prestressing forces and indirect effects due to the rheological properties of materials and uneven precipitation. The variability of the load due to its own weight is influenced by the uncertainties in size, density, additional loads from the nodes and connections of the structural elements, possible changes in the process of reconstruction and / or repair, and the level of construction works quality control. The most valid can take into account the influence of uncertainty in size and density.

Dynamic loads are loads that depend on time and space, sometimes representing a random function. Payloads, snow loads and wind are referred to dynamic loads.

Useful loads are loads that represent a kind of temporary loads, the nature of which is related to the operational (functional) designation of building structures.

Snow loads are loads that depend on the amount of snow. To describe and create a probabilistic model of snow load, statistical parameters of the snow load on the earth's surface and the statistical parameters of the model error, the coefficients of the "transition" from the load on the surface of the earth to the snow on the surface are needed. Coefficients of "transition" take into account: the form of the roof; snow poured from inclined surfaces; snow transport, resulting in uneven deposition over the surface of the cover and the removal of some of the snow that has fallen from the cover; melting snow on the heat-dissipating coatings heated in winter. To approximate the snow load, the first limiting Gumbel distribution, the lognormal distribution, and the Weibull distribution are most widely used. To analyze the reliability of structures characterized by very small probability values, the use of the Gumbel law is safer. The error in the model of this type of load is determined by the variability of the "transition" coefficients. The statistical parameters of coefficients from the load on the surface of the earth to the snow load on the surface have not been sufficiently studied. In most works, probability models of coefficients are adopted according to the recommendations of the Joint Committee of Structural Safety (JCSS). The recommended probabilistic models of snow load are presented in Table 1.

Table 1. – Recommended probabilistic models of snow load

Variable	distribution	μ_x/S_k	V_x
Snow load on the surface of the earth by districts: I II III IV	Gumbel	 0,87 1,01 1,02 1,10	 0,22 0,21 0,2 0,2
Generalized (averaged) model of snow load on the surface of the earth		1,02 – 1,04	0,20 – 0,21
Snow load error	Logarifically normal	1,00	0,1 – 0,15

Wind loads. To describe wind impact models, you need:

- Statistical characteristics of the actual wind speed;
- Basic high-speed head wind;
- Coefficients of "transition" from the base wind speed to the wind profile;
- Coefficients of "transition" from wind speed to wind impact on the environment;
- Errors in the models of the determination of wind influence.

The recommended probabilistic models of wind impact are presented in Table 2.

Table. – Recommended probabilistic models of wind load

Variable	Distribution	μ_x/S_k	V_x
Velocity pressure	Gumbel	1,00 – 1,04	0,13 – 0,15
Wind impact model error	Logarifically normal	0,7 – 0,8	0,28 – 0,3
Wind impact (include model error)	Gumbel	0,7 – 0,8	0,31 – 0,34

With these loads in mind, you can perform the task of constructing a probabilistic model. However, this task must be divided into two important subtasks:

1) Construction of a probabilistic model for modeling based on the results of a technical condition survey.
2) Construction of a probabilistic model at the design stage.

When carrying out the first subtask, the data on the variability of the parameters of materials, loads, structural elements and structures as a whole will be obtained as a result of statistical processing of a small amount of measurement data.

When performing the second, for modeling probabilistic characteristics, large volumes of data on variability will be used, which are formed at the stage of production and construction.

When determining the parameters based on the results of the survey, the sampling data is processed. Each sample value is considered as an independent random variable having a corresponding distribution law with the same parameters.

To perform statistical processing in constructing a probabilistic model of a structural element, it is necessary:

1) Obtain the measurement data;
2) Verify the consistency of the data obtained by a certain distribution law;
3) Identify the distribution parameters of the general population from the received sample data with a given security. Identification is performed in accordance with the chosen distribution law;
4) Determine the regulatory characteristics from the identified distributions.

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