Remark 3. In [5], by the same method of a suitable representation for the linear differential operator, under some conditions on a_j, $j = 0, \ldots, n-1$, a criterion is obtained for equation (1) to have a solution equivalent at infinity to any non-zero constant, and an oscillatory criterion was proved.

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A FORMULA FOR THE BOHL EXPONENT OF DISCRETE TIME-VARYING SYSTEMS

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It is known that the stability of linear time-varying systems is not determined by the position of the spectra of the coefficient matrices. There are examples of continuous-time systems, the coefficients of which have spectra lying in the left half-plane, and these systems are not stable, and conversely all matrices of a stable system may have spectra of the coefficients lying in the right half-plane (see e.g. [6], p. 257). Similarly, one can give examples of stable, asymptotically stable, and even uniformly exponentially stable discrete-time systems with time-varying coefficients, whose coefficient matrices have eigenvalues outside the unit circle, as well as unstable systems with matrices which all eigenvalues lying inside the unit circle. However, if the coefficient matrices of a discrete time-varying system have spectra in the unit circle, stability can be guaranteed by a sufficiently slow variation of the coefficients. This is the basic idea behind the so-called freezing method that was, for discrete-time systems, for the first time described in [2]. A comprehensive description of the results obtained with this technique is provided in Section 10.1 of [4] (see also [3], [5] and the references therein).

Uniform exponential stability of a linear system is characterized by the Bohl exponent. A system is uniformly asymptotically stable if and only if the Bohl exponent is negative ([6], Theorem 3.3.15). The above-mentioned lack of dependence between the spectra of the coefficients and the stability causes that in general it is not possible to give the formula for the Bohl exponent expressed by the eigenvalues of the coefficients. However, as noted by V. M. Millionschikov in [8] and by J. Daleckii and M.G. Krein in the monograph [1, p. 200], such a formula can be given for continuous-time systems with weak variation by Persidskii (see also [7], Section 3.6). The main result of this note is to provide such a formula for discrete systems. On the basis of this formula, we will obtain the necessary and sufficient conditions for uniform exponential stability expressed by the eigenvalues of the matrix of coefficients.

The next theorem contains the main result of this note.

Theorem. *Suppose that for system*

$$x(n+1) = A(n)x(n), \ n \in \mathbb{N}, \tag{1}$$

where $A = (A(n))_{n\in\mathbb{N}}$ *is a sequence of invertible* d *by* d *matrices such that*

$$\sup_{n\in\mathbb{N}} \max\left\{\|A(n)\|, \left\|A^{-1}(n)\right\|\right\} < \infty$$

we have

$$\lim_{n\to\infty} \|A(n+1) - A(n)\| = 0,$$

then

$$\Omega(A) := \limsup_{m,n-m\to\infty} \frac{1}{n-m} \ln \|\Phi_A(n,m)\| = \limsup_{n\to\infty} \ln \lambda\left(A\left(n\right)\right),$$

where $\left(\Phi_A\left(n,m\right)\right)_{n,m\in\mathbb{N}}$ *the transition matrix of system (1) and* $\lambda\left(A\right)$ *the greatest absolute value of the eigenvalues of matrix* A*. In particular system (1) is uniformly asymptotically stable if and only if* $\limsup_{n\to\infty} \ln \lambda\left(A\left(n\right)\right) < 0$.

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NON-AUTONOMOUS SYSTEMS AND TIME-SCALE DYNAMICS: STABILITY AND SHADOWING

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We consider a system on a time scale

$$x^{\Delta} = f(t,x), \quad x \in \mathbb{R}^n, \quad t \in \mathbb{T}, \tag{1}$$

where the time scale $\mathbb{T}$ is an unbounded closed subset of $\mathbb{R}$ while the derivative Δ is defined as follows.