conditions for uniform exponential stability expressed by the eigenvalues of the matrix of coefficients.

The next theorem contains the main result of this note.

**Theorem.** *Suppose that for system*

$$x(n+1) = A(n)x(n), \;\; n \in \mathbb{N}, \tag{1}$$

*where* $A = (A(n))_{n\in\mathbb{N}}$ *is a sequence of invertible* $d$ *by* $d$ *matrices such that*

$$\sup_{n\in\mathbb{N}} \max\left\{\|A(n)\|, \left\|A^{-1}(n)\right\|\right\} < \infty$$

*we have*

$$\lim_{n\to\infty} \|A(n+1) - A(n)\| = 0,$$

*then*

$$\Omega(A) := \limsup_{m,n-m\to\infty} \frac{1}{n-m} \ln \|\Phi_A(n,m)\| = \limsup_{n\to\infty} \ln \lambda\left(A\left(n\right)\right),$$

*where* $\left(\Phi_A\left(n,m\right)\right)_{n,m\in\mathbb{N}}$ *the transition matrix of system (1) and* $\lambda\left(A\right)$ *the greatest absolute value of the eigenvalues of matrix* $A$*. In particular system (1) is uniformly asymptotically stable if and only if* $\limsup\limits_{n\to\infty} \ln \lambda\left(A\left(n\right)\right) < 0$.

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## NON-AUTONOMOUS SYSTEMS AND TIME-SCALE DYNAMICS: STABILITY AND SHADOWING

**S.G. Kryzhevich**

We consider a system on a time scale

$$x^{\Delta} = f(t,x), \quad x \in \mathbb{R}^n, \quad t \in \mathbb{T}, \tag{1}$$

where the time scale $\mathbb{T}$ is an unbounded closed subset of $\mathbb{R}$ while the derivative $\Delta$ is defined as follows.

Given $t \in \mathbb{T}$ we set $t^+ = \inf\{\tau \in \mathbb{T} : \tau > t\}$. Then

$$x^\Delta(t) = \frac{x(t^+) - x(t)}{t^+ - t} \quad \text{if} \qquad t^+ > t$$

and

$$x^\Delta(t) = \lim_{\mathbb{T} \ni \tau \to t+0} \frac{x(\tau) - x(t)}{\tau - t} \qquad \text{if} \quad t^+ = t.$$

Theory of time-scale dynamical systems is quite well-developed, see [1] and references therein. This is an effective approach to study numerical methods of non-uniform steps or systems with strong nonlinearities.

**Definition.** We say that the system (1) is *structurally stable* if all solutions are unique and for any $\varepsilon > 0$ there exists a $\delta > 0$ such that for any $g(t,x) : |g(t,x)| < \delta,\ |g'_x(t,x)| < \delta$ and any $t_0 \in \mathbb{T}$ there is a homeomorphism $h$ of the space $\mathbb{R}^n$ such that

$$|\varphi_f(t,x_0) - \varphi_{f+g}(t,h(x_0))| < \varepsilon$$

for any $x_0 \in \mathbb{R}^n$, $t \in \mathbb{T}$. Here $\varphi_f(t,x_0)$ and $\varphi_{f+g}(x_0)$ are solutions of systems (1) and $x^\Delta = f(t,x) + g(t,x)$ with initial conditions $x(t_0) = x_0$.

For systems of ordinary differential equations, conditions for global structural stability were obtained in [1] and [2]. It was proved that a system is structurally stable if its linearizations are uniformly hyperbolic on families of segments.

We formulate and prove an analog of this statement for time scale systems. Although the result is very similar to that for ordinary differential equations, the proof for the time scale case is significantly different. Besides, the derivatives of $f$ and the so-called graininess function $\mu$ (the size of 'holes' of the time scale) must meet some specific requirements.

To prove the claimed result we need to use specific approaches of time scale systems theory [3]. Remarkably, the classical results for structural stability of autonomous systems of ODEs, obtained by C. Robinson [4], are, in general, non-applicable for systems on time scales (even for the autonomous ones).

The problems of shadowing and inverse shadowing for the considered system and related results for convergence of numerical methods will also be studied.

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