12. Стельмашук Н. Т. *Интегральное представление решений одной системы уравнений в частных производных* // Смоленский матем. сборник. 1970. Т. 3. В. 23. С. 33–38.

BOUNDARY VALUE PROBLEM FOR THE AIRY EQUATION ON LADDER TYPE METRIC GRAPH

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We consider Airy type evolution equation on ladder type metric graph (fig. 1).

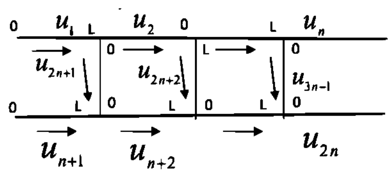

Fig. 1. Ladder type metric graph.

The bonds of the graph denoted by $B_j,\ \ j=\overline{1,3n-1},$ as it is show n in fig. 1.

$$\left(\frac{\partial}{\partial t}-\frac{\partial^3}{\partial x_j^3}\right)u_j(x_j,t)=f_j(x,t),\quad t>0,\quad x_j\in B_j,\quad j=\overline{1,3n-1}. \tag{1}$$

$$b_k u_k(L,t)=a_{k+1}u_{k+1}(0,t)=a_{2n+k}u_{2n+k}(0,t),$$

$$u_1(0,t)=\phi_1(t),u_n(L,t)=\phi_2(t), \tag{2}$$

$$d_k u_k(L,t)=c_{k+1}u_{(k+1)x}(0,t)=c_{2n+k}u_{(2n+k)x}(0,t),$$

$$u_{1x}(0,t)=\delta_1(t), \tag{3}$$

$$\frac{1}{b_k}u_{kxx}(L,t)=\frac{1}{a_{k+1}}u_{(k+1)xx}(0,t)+\frac{1}{a_{2n+k}}u_{(2n+k)xx}(0,t), \tag{4}$$

$$b_{n+k}u_{n+k}(L,t)=a_{n+k+1}u_{n+k+1}(0,t)=a_{2n+k}u_{2n+k}(L,t),$$

$$u_{n+1}(0,t)=\phi_3(t),u_{2n}(L,t)=\phi_4(t), \tag{5}$$

$$c_{n+k+1}u_{(n+k+1)x}(0,t)=d_{2n+k}u_{(2n+k)x}(L,t)+d_{n+k+1}u_{(n+k+1)x}(L,t),$$

$$u_{(n+1)x}(0,t)=\delta_2(t), \tag{6}$$

$$\frac{1}{b_{n+k}}u_{(n+k)xx}(L,t)=\frac{1}{a_{n+k+1}}u_{(n+k+1)xx}(0,t)+\frac{1}{a_{2n+k}}u_{(2n+k)xx}(L,t), \tag{7}$$

for $0<t<T$, $T=$ const. Furthermore, we assume that the functions $f_j(x,t)$, $j=\overline{1,3n-1},$ are smooth enough hand bounded. The initial conditions are given by:

$$u_j(x,0)=u_{0,j}(x),\quad x\in\overline{B_j},\quad j=\overline{1,3n-1}. \tag{8}$$

It should be noted that the above vertex conditions are not the only possible ones. The main motivation for our choice is caused by the fact that they guarantee uniqueness of the solution and, if the solutions decay (to zero) at infinity, the norm (energy) conservation.

Refrences

1. Sobirov Z. A., Akhmedov M. I., Uecker H. *Cauchy problem for the linearized KdV equation on general metric star graphs.*

2. Noja D. *Nonlinear Schrodinger equations on graphs: recent results and open problems* // Phil. Trans. Roy Soc. A. 2014. 372, 20130002.

BLOW-UP PROBLEM FOR SEMILINEAR PARABOLIC EQUATION WITH GENERAL NONLINEARITIES IN EQUATION AND BOUNDARY CONDITION

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We consider the global solvability and blow-up in finite time for semilinear heat equation

$$u_t = \Delta u + \alpha(t) f(u) \text{ for } x \in \Omega, \ \ t > 0, \tag{1}$$

with nonlinear boundary condition

$$\frac{\partial u(x,t)}{\partial \nu} = \beta(t) g(u) \text{ for } x \in \partial\Omega, \ \ t > 0, \tag{2}$$

and initial datum

$$u(x,0) = u_0(x) \text{ for } x \in \Omega, \tag{3}$$

where Ω is a bounded domain in $\mathbb{R}^n$ for $n \geqslant 1$ with smooth boundary $\partial\Omega$, ν is the unit exterior normal vector on the boundary $\partial\Omega$. Here $f(u)$ and $g(u)$ are nonnegative continuous functions for $u \geqslant 0$, $\alpha(t)$ and $\beta(t)$ are nonnegative continuous functions for $t \geqslant 0$, $u_0(x) \in C^1(\overline{\Omega})$, $u_0(x) \geqslant 0$ in $\overline{\Omega}$ and satisfies boundary condition (2) as $t = 0$. We will consider nonnegative classical solutions of (1)–(3).

We prove several blow-up results for (1)–(3).

Theorem 1. *Let $g(s)$ be a nondecreasing positive function for $s > 0$ such that*

$$\int^{+\infty} \frac{ds}{g(s)} < +\infty$$

and

$$\int_0^{+\infty} \beta(t)\, dt = +\infty.$$

Then any nontrivial nonnegative solution of (1)–(3) blows up in finite time.

Theorem 2. *Let $f(s) > 0$ for $s > 0$,*

$$\int^{+\infty} \frac{ds}{f(s)} < +\infty$$

and

$$\int_0^{+\infty} \alpha(t)\, dt = +\infty.$$