Refrences

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BLOW-UP PROBLEM FOR SEMILINEAR PARABOLIC EQUATION WITH GENERAL NONLINEARITIES IN EQUATION AND BOUNDARY CONDITION

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We consider the global solvability and blow-up in finite time for semilinear heat equation

$$u_t = \Delta u + \alpha(t) f(u) \text{ for } x \in \Omega, \ \ t > 0, \tag{1}$$

with nonlinear boundary condition

$$\frac{\partial u(x,t)}{\partial \nu} = \beta(t) g(u) \text{ for } x \in \partial\Omega, \ \ t > 0, \tag{2}$$

and initial datum

$$u(x,0) = u_0(x) \text{ for } x \in \Omega, \tag{3}$$

where Ω is a bounded domain in $\mathbb{R}^n$ for $n \geqslant 1$ with smooth boundary $\partial\Omega$, ν is the unit exterior normal vector on the boundary $\partial\Omega$. Here $f(u)$ and $g(u)$ are nonnegative continuous functions for $u \geqslant 0$, $\alpha(t)$ and $\beta(t)$ are nonnegative continuous functions for $t \geqslant 0$, $u_0(x) \in C^1(\overline{\Omega})$, $u_0(x) \geqslant 0$ in $\overline{\Omega}$ and satisfies boundary condition (2) as $t = 0$. We will consider nonnegative classical solutions of (1)–(3).

We prove several blow-up results for (1)–(3).

Theorem 1. *Let $g(s)$ be a nondecreasing positive function for $s > 0$ such that*

$$\int^{+\infty} \frac{ds}{g(s)} < +\infty$$

and

$$\int_0^{+\infty} \beta(t)\, dt = +\infty.$$

Then any nontrivial nonnegative solution of (1)–(3) blows up in finite time.

Theorem 2. *Let $f(s) > 0$ for $s > 0$,*

$$\int^{+\infty} \frac{ds}{f(s)} < +\infty$$

and

$$\int_0^{+\infty} \alpha(t)\, dt = +\infty.$$

Then any nontrivial nonnegative solution of (1)–(3) blows up in finite time.

To formulate global existence result for problem (1)–(3) we suppose:

$$f(s) \text{ is a nonnegative locally Hölder continuous function for } s \geqslant 0, \tag{4}$$

$$\text{there exists } p > 0 \text{ such that } f(s) \text{ is a positive nondecreasing function for } s \in (0, p), \tag{5}$$

$$\int_0 \frac{ds}{f(s)} = +\infty, \quad \lim_{s \to 0} \frac{g(s)}{s} = 0, \tag{6}$$

$$\int_0^{+\infty} \big(\alpha(t) + \beta(t)\big)\, dt < +\infty \tag{7}$$

and there exist positive constants γ, t_0 and K such that $\gamma > t_0$ and

$$\int_{t-t_0}^{t} \frac{\beta(\tau) d\tau}{\sqrt{t-\tau}} \leqslant K \quad \text{for } t \geqslant \gamma. \tag{8}$$

Theorem 3. *Let (4)–(8) hold. Then problem (1)–(3) has bounded global solution for small initial datum.*

The results of the talk have been published in [1].

References

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CLASSICAL SOLUTION OF THE INITIAL-VALUE PROBLEM FOR A ONE-DIMENSIONAL QUASILINEAR WAVE EQUATION

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In this report we shall consider the question of global solvability in $[0, \infty) \times \mathbb{R}$ of the initial-value problem

$$\begin{cases} \partial_t^2 u(t,x) - a^2 \partial_x^2 u(t,x) + f\big(t, x, u(t,x), \partial_t u(t,x), \partial_x u(t,x)\big) = F(t,x), & (t,x) \in (0,\infty) \times \mathbb{R}, \\ u(0,x) = \varphi(x), \quad \partial_t u(0,x) = \psi(x), & x \in \mathbb{R}, \end{cases} \tag{1}$$

where $a \in (0, \infty)$, φ and ψ are some real-valued functions defined on the real axis.

Theorem 1. *Assume $\varphi \in C^2(\mathbb{R})$, $\psi \in C^1(\mathbb{R})$, $F \in C^1([0,\infty) \times \mathbb{R})$, $f \in C^1([0,\infty) \times \mathbb{R}^4)$ and f is Lipschitz continuous in the three last variables. Then there exists a unique classical solution u of the initial-value problem (1).*

Sketch of the proof. We will look for a solution u having the form $u = w + v$ where v solves the homogeneous wave equation

$$\begin{cases} \partial_t^2 v(t,x) - a^2 \partial_x^2 v(t,x) = 0, & (t,x) \in (0,\infty) \times \mathbb{R}, \\ v(0,x) = \varphi(x), \ \ \partial_t v(0,x) = \psi(x), & x \in \mathbb{R}, \end{cases}$$