Refrences

1. Roberts P.H. *Introduction to Magnetohydrodynamics.* Longsmans, London, 1967
2. Sommeria J., Moreau R. *Why, how and when, MHD turbulence becomes two-dimensional* // J. Fluid Mech. 2003. № 118. P. 507–518.
3. Foias C., Manley O., Rosa R., Temam R. *Navier-Stokes Equations and Turbulence.* Cambridge University Press, 2001.
4. Dymkou V., Potherat A. *Spectral method based on the least dissipative modes for wall bounded MHD flows* // Theor. Comput. Fluid Dyn. 2009. № 23. P. 535. https://doi.org/10.1007/s00162-009-0159-9

MULTI-DIMENSIONAL GENERAL INTEGRAL TRANSFORMATION WITH SPECIAL FUNCTIONS IN THE WEIGHTED SPACE OF SUMMABLE FUNCTIONS

S.M. Sitnik, O.V. Skoromnik, S.A. Shlapakov

Multidimensional integral transform

$$\big(\mathrm{K}f\big)(\mathbf{x}) = \overline{h}\mathbf{x}^{1-(\overline{\lambda}+1)/\overline{h}}\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}\,\mathbf{x}^{(\overline{\lambda}+1)/\overline{h}}\int\limits_{\mathbf{0}}^{\infty}\mathrm{k}[\mathbf{x}\mathbf{t}]f(\mathbf{t})\,\mathrm{d}\mathbf{t}\quad(\mathbf{x}>0) \tag{1}$$

is studied. Here (see, for example, [1] Section 28.4; [2], ch. 1; [3], [4])

$$\mathbf{x}=(x_1,x_2,...,x_n)\in\mathbb{R}^n;\quad \mathbf{t}=(t_1,t_2,...,t_n)\in\mathbb{R}^n,$$

$\mathbb{R}^n$ be the n-dimensional Euclidean space; $\mathbf{x}\cdot\mathbf{t}=\sum\limits_{n=1}^{n}x_nt_n$ denotes their scalar product; in particular, $\mathbf{x}\cdot\mathbf{1}=\sum\limits_{n=1}^{n}x_n$ for **1**= (1,1,...,1). The expression $\mathbf{x}>\mathbf{t}$ means that

$$x_1>t_1,\quad x_2>t_2,\quad ...,\quad x_n>t_n,$$

the nonstrict inequality $\geqslant$ has similar meaning;

$$\int\limits_{\mathbf{0}}^{\infty}=\int\limits_{0}^{\infty}\int\limits_{0}^{\infty}\cdots\int\limits_{0}^{\infty};$$

by $\mathbb{N}=\{1,2,...\}$ we denote the set of positive integers,

$$\mathrm{N}_0=\mathbb{N}\cup\{0\},\quad \mathrm{N}_0^n=\mathrm{N}_0\times\mathrm{N}_0\times...\times\mathrm{N}_0.$$

$$\mathbf{k}=(k_1,k_2,...,k_n)\in\mathrm{N}_0^n=\mathrm{N}_0\times...\times\mathrm{N}_0\ \ (k_i\in\mathrm{N}_0,\ \ i=1,2,...,n)$$

is a multi-index with $\mathbf{k}!=k_1!\cdots k_n!$ and $|\mathbf{k}|=k_1+k_2+...+k_n$.

$$\mathbb{R}^n_+=\{\mathbf{x}\in\mathbb{R}^n,\ \mathbf{x}>0\};$$

for $l=(l_1,l_2,...,l_n)\in\mathbb{R}^n_+$

$$\mathbf{D}^l=\frac{\partial^{|l|}}{(\partial x_1)^{l_1}\cdots(\partial x_n)^{l_n}};\qquad \mathbf{dt}=dt_1\cdot dt_2\cdots dt_n;$$

$$\mathbf{t}^l = t^{l_1}t^{l_2}\cdots t^{l_n}; \quad f(\mathbf{t}) = f(t_1, t_2, ..., t_n).$$

Let $\mathbb{C}^n$ $(n \in \mathbb{N})$ be the n-dimensional space of n complex numbers

$$z = (z_1, z_2, \ldots, z_n) \quad (z_j \in \mathbb{C},\ j = 1, 2, \cdots, n).$$

$$\overline{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_n) \in \mathbb{C}^n; \quad \overline{h} = (h_1, h_2, ..., h_n), \quad h_j \in \mathbb{R} \setminus \{0\}, \quad j = 1, 2, ..., n;$$

$$\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}} = \frac{d}{dx_1 \cdot dx_2 \cdots dx_n}.$$

We introduce the function in the kernel $\mathrm{k}[\mathbf{xt}] = \mathrm{k}[x_1t_1] \cdot \mathrm{k}[x_2t_2] \cdots \mathrm{k}[x_nt_n]$, which is the product of some one type special functions.

Our paper is devoted to the study of tranform (1) $\mathrm{K}f$ in the weighted spaces $\mathfrak{L}_{\overline{\nu}, \overline{r}}$ summabe functions $f(\mathbf{x}) = f(x_1, x_2, ..., x_n)$ on

$$\mathbb{R}^{\mathbf{n}}_+ = \{\mathbf{x} : \mathbf{x} \in \mathbb{R}^n \,|\, x_1 > 0,\ x_2 > 0,\ ...,\ x_n > 0\},$$

such that

$$\|f\|_{\overline{\nu},\overline{r}} = \Big\{ \int\limits_{R^1_+} x_n^{\nu_n \cdot r_n - 1} \Big\{ \cdots \Big\{ \int\limits_{R^1_+} x_2^{\nu_2 \cdot r_2 - 1}$$

$$\times \Big[\int\limits_{R^1_+} x_1^{\nu_1 \cdot r_1 - 1} |f(x_1, ..., x_n)|^{r_1} dx_1 \Big]^{r_2/r_1} dx_2 \Big\}^{r_3/r_2} \cdots \Big\}^{r_n/r_{n-1}} dx_n \Big\}^{1/r_n} < \infty \tag{2}$$

$\big(\overline{r} = (r_1, r_2, ..., r_n) \in \mathbb{R}^n,\ 1 \leqslant \overline{r} < \infty,\ \overline{\nu} = (\nu_1, \nu_2, ..., \nu_n) \in \mathbb{R}^n,\ \nu_1 = \nu_2 = ... = \nu_n\big)$.

The functional and compositional properties of the integral transformation (1) in spaces $\mathfrak{L}_{\overline{\nu}, \overline{2}}$ $\big(\overline{2} = (2, 2, ..., 2),\ \overline{\nu} = (\nu_1, \nu_2, ..., \nu_n) \in \mathbb{R}^n,\ \nu_1 = \nu_2 = ... = \nu_n\big)$ were studied in [3]. We continue this research. The scheme of study is similar to the process of constructing the theory of the H-transformation, in which the central place is given to the questions of bounded and one-to-one action of the corresponding integral operator in spaces of integrable functions with weight concentrated at zero and at infinity. Theory of the considered integral transformation (1) in weighted spaces $\mathfrak{L}_{\overline{\nu}, \overline{r}}$ of summable functions is constructed. Mapping properties such as the boundedness, the range, the representation and the inversion of the considered transform (1) in the weighted space (2) are established.

Refrences

1. Samko S. G., Kilbas A. A., Marichev O. I. *Fractional Integrals and Derivatives: Theory and Applications.* London: Gordon and Breach Science Publishers, 1993.

2. Kilbas A. A., Srivastava H. M., Trujillo J. J. *Theory and applications of fractional differential equations.* Amsterdam: Elsevier Science, 2006.

3. Sitnik S. M, Skoromnik O. V., Shlapakov S. A. *Multidimensional general integral transformation with special functions in the kernel* // Bulletin of the Vitebsk State university. 2019. № 3(104). P. 18–27.

4. Sitnik S. M., Skoromnik O. V. *One-dimensional and multi-dimensional integral transforms of Buschman–Erdelyi type with Legendre Functions in kernels* // Transmutation Operators and Applications. Trends in Mathematics. 2020. P. 293–319.