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GEOMETRIC APPROACH TO THE STUDY NAVIER-STOKES EQUATIONS

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Theorem 1. *The* $14D$ *Riemann metric in local coordinates*

$$\vec{x} = (x,\, y,\, z,\, t,\, \eta,\, \rho,\, m,\, u,\, v,\, w,\, p,\, \xi,\, \chi,\, n):$$

$$ds^2 = 2\,dxdu + 2\,dydv + 2\,dzdw + \Big(-W(\vec{x},t)w - V(\vec{x},t)v - U(\vec{x},t)u\Big)dt^2 +$$
$$+\Big(-U(\vec{x},t)p - u\,(U(\vec{x},t))^2 - uP(\vec{x},t) + w\mu\,\frac{\partial}{\partial z}U(\vec{x},t) - wU(\vec{x},t)W(\vec{x},t)\Big)d\eta^2 +$$
$$+\Big(v\mu\,\frac{\partial}{\partial y}U(\vec{x},t) - vU(\vec{x},t)V(\vec{x},t) + u\mu\,\frac{\partial}{\partial x}U(\vec{x},t)\Big)d\eta^2 + 2\,d\eta d\xi + 2\,d\rho d\chi + 2\,dmdn +$$
$$+\Big(-V(\vec{x},t)p - vP(\vec{x},t) - v\,((\vec{x},t))^2 - V(\vec{x},t)W(\vec{x},t)w + v\mu\,\frac{\partial}{\partial y}V(\vec{x},t) - uU(\vec{x},t)V(\vec{x},t)\Big)d\rho^2 +$$
$$+\Big(u\mu\,\frac{\partial}{\partial x}V(\vec{x},t)\Big)d\rho^2 + \Big(-uU(\vec{x},t)W(\vec{x},t) - w\,(W(\vec{x},t))^2 - wP(\vec{x},t) + w\mu\,\frac{\partial}{\partial z}W(\vec{x},t)\Big)dm^2 +$$
$$+\Big(v\mu\,\frac{\partial}{\partial y}W(\vec{x},t) - vV(\vec{x},t)W(\vec{x},t) + u\mu\,\frac{\partial}{\partial x}W(\vec{x},t) - W(\vec{x},t)p\Big)dm^2 \qquad (1)$$

is the Ricci-flat,

$$R_{44} = U_x + V_y + W_z = 0, \quad R_{55} = 0, \quad R_{66} = 0, \quad R_{77} = 0$$

on solutions of Navier-Stokes system of equations

$$\frac{\partial}{\partial t}\vec{Q}(\vec{x},t) + (\vec{Q}(\vec{x},t)\cdot\vec{\nabla})\vec{Q}(\vec{x},t) - \mu\,\Delta\vec{Q}(\vec{x},t) + \vec{\nabla}P(\vec{x},t) = 0, \quad \vec{\nabla}\cdot\vec{Q}(\vec{x},t) = 0, \qquad (2)$$

where $\vec{Q}(\vec{x},t) = [U(\vec{x},t), V(\vec{x},t), W(\vec{x},t)]$ *are the components of velocity and* $P(\vec{x},t)$ *is pressure of liquid* (see e.g. [1-2]).

To obtain the metric (1) presentation the NS-system of equations in the form of laws conservations

$$U_t + (U^2 - \mu U_x + P)_x + (UV - \mu U_y)_y + (UW - \mu U_z)_z = 0,$$

$$V_t + (V^2 - \mu V_y + P)_y + (UV - \mu V_x)_x + (VW - \mu V_z)_z = 0,$$
$$W_t + (W^2 - \mu W_z + P)_z + (UW - \mu W_x)_x + (VW - \mu W_y)_y = 0,$$
$$(U_x + V_y + W_z) = 0,$$

is used.

The metric (1) belongs to the class of the Riemann spaces with vanishing scalar Invariants and part of its geodesics with respect to the coordinates $\eta, \rho, m, \xi, \chi, n$ has form of the equations

$$\ddot{\eta} = 0, \quad \ddot{\rho} = 0, \quad \ddot{m} = 0, \quad \ddot{\xi} = 0, \quad \ddot{\chi} = 0, \quad \ddot{n} = 0.$$

On the base of solutions of equations for the Killing vectors of the metric

$$K_{i,j} + K_{j,i} - 2\Gamma^k_{ij} K_k = 0, \quad \text{or} \quad K^k g_{ij,k} + g_{ik} K^k,\ j + g_{jk} K^k,\ i = 0, \tag{3}$$

a new examples of reductions and solutions of the system (2) are constructed.

Properties of the Lie derivative for the connection coefficients of the metric (1) and the vector field of the form $u^i = g^i_k v^k$

$$u^i_{j,k} + u^n \Gamma^i_{jk},_n + u^n_{,j} \Gamma^i_{nk} + u^n_{,k} \Gamma^i_{jn} - u^n_{,n} \Gamma^i_{jk} = 0,$$

where Γ^i_{jk}-are the coefficients of connection of the metric (1) with the aim of constructing new examples of solutions to the system (2) are discussed.

Another possibility for studying the properties of the NS system by the geometric method is the use of differential Beltrami parameters of the metric (1)

$$\Delta_2(f) = g^{ij} \frac{\partial^2 f}{\partial x_i, x_j} - \Gamma^k_{ij} \frac{\partial f}{\partial x_k}.$$

As example, in particular case

$$f = \psi(x,\ y,\ z,\ t,\ 0,\ 0,\ 0,\ u,\ v,\ w,\ p,\ 0,\ 0,\ 0),$$

from solutions of the linear equation with variable coefficients

$$\Delta_2(f) = 0$$

the relation

$$\big(U(\vec{x},t) - W(\vec{x},t)\big) P(\vec{x},t)/\mu = U(\vec{x},t) \frac{\partial}{\partial z} U(\vec{x},t) - W(\vec{x},t) \frac{\partial}{\partial x} V(\vec{x},t) -$$
$$-W(\vec{x},t) \frac{\partial}{\partial x} U(\vec{x},t) - W(\vec{x},t) \frac{\partial}{\partial x} W(\vec{x},t) + \frac{\partial}{\partial z} V(\vec{x},t) U(\vec{(}x,t)) + \frac{\partial}{\partial z} W(\vec{x},t) U(\vec{x},t),$$

between velocity and pressure can be derived and that can be applied to the studying properties of solutions of the system (2).

Theorem 2. *From auxiliary overdetermined linear system of partial differential equations for the function* $\Phi(\vec{x},t)$

$$\frac{\partial^2}{\partial x_i, \partial x_j} \Phi(\vec{x},t) = A(\vec{x},t) \frac{\partial}{\partial x_i} \Phi(\vec{x},t) + B(\vec{x},t) \frac{\partial}{\partial x_j} \Phi(\vec{x},t) +$$
$$+ C(\vec{x},t) \frac{\partial}{\partial x_k} \Phi(\vec{x},t) + E(\vec{x},t) \Phi(\vec{x},t); \tag{4}$$

with coefficients depended on the velocities $Q(\vec{x},t)$, the conditions and equations like of the form

$$\frac{\partial}{\partial y}U(\vec{x},t) =$$

$$= \frac{\frac{\partial^2}{\partial x\partial z}H(\vec{x},t))\frac{\partial}{\partial y}\Phi(\vec{x},t)+\frac{\partial^3}{\partial y\partial x\partial z}H(\vec{x},t)\Phi(\vec{x},t) -\frac{\partial^2}{\partial x\partial y}\Phi(\vec{x},t)}{-\frac{\partial^2}{\partial x\partial z}}H(\vec{x},t)\Phi(\vec{x},t)+\frac{\partial}{\partial x}\Phi(\vec{x},t),$$

$$\frac{\partial}{\partial z}U(\vec{x},t) = -\frac{\frac{\partial^2}{\partial x\partial z}H(\vec{x},t)\frac{\partial}{\partial z}\Phi(\vec{x},t) + \Phi(\vec{x},t)\frac{\partial^3}{\partial z\partial x\partial z}H(\vec{x},t) - \frac{\partial^2}{\partial x\partial z}\Phi(\vec{x},t)}{-\frac{\partial^2}{\partial x\partial z}H(\vec{x},t)\Phi(\vec{x},t) + \frac{\partial}{\partial x}\Phi(\vec{x},t)}$$

and the equation

$$-\mu\frac{\partial^4}{\partial x^3\partial z}H(\vec{x},t)-\mu\frac{\partial^4}{\partial y^2\partial x\partial z}H(\vec{x},t)-\mu\frac{\partial^4}{\partial z^2\partial x\partial z}H(\vec{x},t)-$$

$$-\frac{\partial^3}{\partial z\partial x\partial z}H(\vec{x},t)\frac{\partial}{\partial y}H(\vec{x},t)-\frac{\partial^3}{\partial z\partial x\partial z}H(\vec{x},t)\frac{\partial^2}{\partial x\partial y}H(\vec{x},t)+$$

$$+\frac{\partial^2}{\partial x\partial z}H(\vec{x},t)\frac{\partial^3}{\partial y\partial x\partial z}H(\vec{x},t)+U(x,y,z,t)\frac{\partial^3}{\partial x^2\partial z}H(\vec{x},t)+$$

$$+\frac{\partial}{\partial y}P(\vec{x},t)+\frac{\partial^3}{\partial x\partial t\partial z}H(\vec{x},t) = 0$$

are performed.

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OF SPECTRA OF THE ENERGY OPERATOR OF THE FOUR-ELECTRON SYSTEMS IN THE HUBBARD MODEL. QUINTET STATE. TWO- AND THREE-DIMENSIONAL CASE

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The structure of essential spectra and discrete spectrum of the energy operator of four-electron systems in the impurity Hubbard model in a quintet state in the one-dimensional case were studied in [1]. We consider the energy operator of four-electron systems in the impurity Hubbard model and investigated the structure of essential spectra and discrete spectrum of the system in the quintet state in two- and three-dimensional case. Hamiltonian of the system has the form

$$H = A\sum_{m,\gamma}a^+_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma}a^+_{m,\gamma}a_{m+\tau,\gamma} + U\sum_{m}a^+_{m,\uparrow}a_{m,\uparrow}a^+_{m,\downarrow}a_{m,\downarrow}+$$

$$+(A_0-A)\sum_{\gamma}a^+_{0,\gamma}a_{0,\gamma} + (B_0-B)\sum_{\tau,\gamma}(a^+_{0,\gamma}a_{\tau,\gamma} + a^+_{\tau,\gamma}a_{0,\gamma}) + (U_0-U)a^+_{0,\uparrow}a_{0,\uparrow}a^+_{0,\downarrow}a_{0,\downarrow}. \quad (1)$$