with coefficients depended on the velocities $Q(\vec{x},t)$, *the conditions and equations like of the form*

$$\frac{\partial}{\partial y}U(\vec{x},t) =$$

$$= \frac{\frac{\partial^2}{\partial x\partial z}H(\vec{x},t))\frac{\partial}{\partial y}\Phi(\vec{x},t)+\frac{\partial^3}{\partial y\partial x\partial z}H(\vec{x},t)\Phi(\vec{x},t) -\frac{\partial^2}{\partial x\partial y}\Phi(\vec{x},t)}{-\frac{\partial^2}{\partial x\partial z}}H(\vec{x},t)\Phi(\vec{x},t)+\frac{\partial}{\partial x}\Phi(\vec{x},t),$$

$$\frac{\partial}{\partial z}U(\vec{x},t) = -\frac{\frac{\partial^2}{\partial x\partial z}H(\vec{x},t)\frac{\partial}{\partial z}\Phi(\vec{x},t) + \Phi(\vec{x},t)\frac{\partial^3}{\partial z\partial x\partial z}H(\vec{x},t) - \frac{\partial^2}{\partial x\partial z}\Phi(\vec{x},t)}{-\frac{\partial^2}{\partial x\partial z}H(\vec{x},t)\Phi(\vec{x},t) + \frac{\partial}{\partial x}\Phi(\vec{x},t)}$$

and the equation

$$-\mu\frac{\partial^4}{\partial x^3\partial z}H(\vec{x},t)-\mu\frac{\partial^4}{\partial y^2\partial x\partial z}H(\vec{x},t)-\mu\frac{\partial^4}{\partial z^2\partial x\partial z}H(\vec{x},t)-$$

$$-\frac{\partial^3}{\partial z\partial x\partial z}H(\vec{x},t)\frac{\partial}{\partial y}H(\vec{x},t)-\frac{\partial^3}{\partial z\partial x\partial z}H(\vec{x},t)\frac{\partial^2}{\partial x\partial y}H(\vec{x},t)+$$

$$+\frac{\partial^2}{\partial x\partial z}H(\vec{x},t)\frac{\partial^3}{\partial y\partial x\partial z}H(\vec{x},t)+U(x,y,z,t)\frac{\partial^3}{\partial x^2\partial z}H(\vec{x},t)+$$

$$+\frac{\partial}{\partial y}P(\vec{x},t)+\frac{\partial^3}{\partial x\partial t\partial z}H(\vec{x},t) = 0$$

are performed.

Aknowledgement. The work is partially supported by NSF.

Refrences

1. Dryuma V. S. *On spaces related to the Navier-Stokes equations* // Buletinul Academiei de Stiinte a Republicii Moldova. Matematica. 2010. V. 3(64). P. 107–110.

2. Dryuma V. S. *The Ricci-flat spaces related to the Navier-Stokes equations* // Buletinul Academiei de Stiinte a Republicii Moldova. Matematica. 2012. V. 2(69). P. 99–102.

3. Dryuma V. S. *The Riemann and Einstein geometries in the theory of ODE's, their applications and all that* // New Trends in Integrability and Partial Solvability. Kluwer Publisher. (A.B. Shabat at al.(eds.)), 2004. P. 115–156.

OF SPECTRA OF THE ENERGY OPERATOR OF THE FOUR-ELECTRON SYSTEMS IN THE HUBBARD MODEL. QUINTET STATE. TWO- AND THREE-DIMENSIONAL CASE

S.M. Tashpulatov, R.T. Parmanova

The structure of essential spectra and discrete spectrum of the energy operator of four-electron systems in the impurity Hubbard model in a quintet state in the one-dimensional case were studied in [1]. We consider the energy operator of four-electron systems in the impurity Hubbard model and investigated the structure of essential spectra and discrete spectrum of the system in the quintet state in two- and three-dimensional case. Hamiltonian of the system has the form

$$H = A\sum_{m,\gamma}a^+_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma}a^+_{m,\gamma}a_{m+\tau,\gamma} + U\sum_m a^+_{m,\uparrow}a_{m,\uparrow}a^+_{m,\downarrow}a_{m,\downarrow}+$$

$$+(A_0-A)\sum_{\gamma}a^+_{0,\gamma}a_{0,\gamma} + (B_0-B)\sum_{\tau,\gamma}(a^+_{0,\gamma}a_{\tau,\gamma}+a^+_{\tau,\gamma}a_{0,\gamma}) + (U_0-U)a^+_{0,\uparrow}a_{0,\uparrow}a^+_{0,\downarrow}a_{0,\downarrow}. \quad (1)$$

Here A (A_0) is the electron energy at a regular (impurity) lattice site; B (B_0) is the transfer integral between electrons (between electron and impurity) in a neighboring sites (we assume that $B>0$, $B_0>0$), $\tau=\pm e_j$, $j=1,2,...,\nu$, where e_j are unit mutually orthogonal vectors, which means that summation is taken over the nearest neighbors, U (U_0) is the parameter of the on-site Coulomb interaction of two electrons, correspondingly in the regular (impurity) lattice site; γ is the spin index, $\gamma=\uparrow$ or $\gamma=\downarrow$, and $a^+_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $m\in Z^\nu$, where Z^ν is a ν-dimensional integer lattice.

It is known that the Hamiltonian H acts in the antisymmetric complex Foc'k space $(\mathcal{H}_{as},(.)_{\mathcal{H}_{as}})$. Suppose that φ_0 is the vacuum vector in the space $\mathcal{H}_{as}$. The quintet state corresponds to the free motion of four electrons over the lattice with the basic functions $q^2_{m,n,k,l\in Z^\nu}=a^+_{m,\uparrow}a^+_{n,\uparrow}a^+_{k,\uparrow}a^+_{l,\uparrow}\varphi_0$. The linear subspace $\mathcal{H}^q_2$, corresponding the quintet state is the set of all vectors of the form $\psi^q_2=\sum_{m,n,k,l\in Z^\nu}f(m,n,k,l)q^2_{m,n,k,l\in Z^\nu}$, $f\in l^{as}_2$, where l^{as}_2 is the subspace of antisymmetric functions in the space $l_2\big((Z^\nu)^4\big)$. We denote by H^q_2 the restriction of operator H to the subspace $\mathcal{H}^q_2$. We let $\varepsilon_1=A_0-A$, $\varepsilon_2=B_0-B$, and $\varepsilon_3=U_0-U$.

Let $\mathcal{F}:l_2((Z^\nu)^4\to L_2((T^\nu)^4\equiv\mathcal{H}^q_2$ be the Fourier transform, where T^ν is the ν-dimensional torus endowed with the normalized Lebesgue measure $d\lambda$, i.e. $\lambda(T^\nu)=1$. We set $\widetilde{H}^q_2=\mathcal{F}H^q_2\mathcal{F}^{-1}$.

Theorem 1. *The Fourier transform of operator H^q_2 is an operator $\widetilde{H}^q_2=\mathcal{F}H^q_2\mathcal{F}^{-1}$ acting in the space $L^{as}_2\big((T^\nu)^4\big)$ by the formula*

$$\widetilde{H}^q_2\,\psi^q_2=h(\lambda,\mu,\gamma,\theta)f(\lambda,\mu,\gamma,\theta)+$$

$$+\varepsilon_1\Big[\int\limits_{T^\nu}f(s,\mu,\gamma,\theta)ds+\int\limits_{T^\nu}f(\lambda,t,\gamma,\theta)dt+\int\limits_{T^\nu}f(\lambda,\mu,k,\theta)dk+\int\limits_{T^\nu}f(\lambda,\mu,\gamma,\xi)d\xi\Big]+$$

$$+2\varepsilon_2\Big[\int\limits_{T^\nu}\sum_{i=1}^{\nu}[\cos\lambda_i+\cos s_i]f(s,\mu,\gamma,\theta)ds+\int\limits_{T^\nu}\sum_{i=1}^{\nu}[\cos\mu_i+\cos t_i]f(\lambda,t,\gamma,\theta)dt+$$

$$+\int\limits_{T^\nu}\sum_{i=1}^{\nu}[\cos\gamma_i+\cos k_i]f(\lambda,\mu,k,\theta)dk+\int\limits_{T^\nu}\sum_{i=1}^{\nu}[\cos\theta_i+\cos\xi_i]f(\lambda,\mu,\gamma,\xi)d\xi\Big],\qquad(2)$$

where $h(\lambda,\mu,\gamma,\theta)=4A+2B\sum_{i=1}^{\nu}[\cos\lambda_i+\cos\mu_i+\cos\gamma_i+\cos\theta_i]$, and L^{as}_2 is the subspace of antisymmetric functions in $L_2\big((T^\nu)^4\big)$.

In the impurity Hubbard model, the spectral properties of the energy operator of four-electron systems are closely related to those of its two-particle subsystems (one-electron systems with impurity). Therefore, we first study the spectrum and localized impurity electron states of the one-electron impurity systems. Subsequently, using tensor products of Hilbert spaces and tensor products of operators in Hilbert spaces and taking into account that the function $f(\lambda,\mu,\gamma,\theta)$ is an antisymmetric function and using of results of investigation of spectra of one-electron systems with impurity, we describe the structure of essential spectrum and discrete spectrum of the energy operator of four electron systems in the impurity Hubbard model in the quintet state.

Theorem 2. *Let $\nu=2$. Then*

a). If $\varepsilon_2=-B$ and $\varepsilon_1<-4B$ (respectively, $\varepsilon_2=-B$ and $\varepsilon_1>4B$), then the essential spectrum of the operator $\widetilde{H}^q_2$ consists of the union of four segments:

$$\sigma_{ess}(\widetilde{H}_2^q) = [4A - 16B, 4A + 16B] \cup [3A - 12B + z, 3A + 12B + z] \cup$$

$$\cup [2A - 8B + 2z, 2A + 8B + 2z] \cup [A - 4B + 3z, A + 4B + 3z],$$

and discrete spectrum of the operator $\widetilde{H}_2^q$ *consists of a single eigenvalue:* $\sigma_{disc}(\widetilde{H}_2^q) = \{4z\}$, *where* $z = A + \varepsilon_1$, *lying the below (respectively, above) of the essential spectrum of the operator* $\widetilde{H}_2^q$.

b). If $\varepsilon_1 < 0$ *and* $\varepsilon_2 = -2B$ *or* $\varepsilon_2 = 0$ *(respectively,* $\varepsilon_1 > 0$ *and* $\varepsilon_2 = -2B$ *or* $\varepsilon_2 = 0$ *), then the essential spectrum of the operator* $\widetilde{H}_2^q$ *consists of the union of four segments:*

$$\sigma_{ess}(\widetilde{H}_2^q) = [4A - 16B, 4A + 16B] \cup [3A - 12B + \widetilde{z}, 3A + 12B + \widetilde{z}] \cup$$

$$\cup [2A - 8B + 2\widetilde{z}, 2A + 8B + 2\widetilde{z}] \cup [A - 4B + 3\widetilde{z}, A + 4B + 3\widetilde{z}],$$

and discrete spectrum of the operator $\widetilde{H}_2^q$ *consists of a single eigenvalue:* $\sigma_{disc}(\widetilde{H}_2^q) = \{4\widetilde{z}\}$, *where* $\widetilde{z}$, *same concrete real number, lying the below (respectively, above) of the essential spectrum of the operator* $\widetilde{H}_2^q$.

c). If $\varepsilon_1 = 0$ *and* $\varepsilon_2 > 0$ *or* $\varepsilon_1 = 0$ *and* $\varepsilon_2 < -2B$, *then the essential spectrum of the operator* $\widetilde{H}_2^q$ *consists of the union of ten segments:*

$$\sigma_{ess}(\widetilde{H}_2^q) = [4A - 16B, 4A + 16B] \cup [3A - 12B + \widetilde{z}_1, 3A + 12B + \widetilde{z}_1] \cup$$

$$\cup [3A - 12B + \widetilde{z}_2, 3A + 12B + \widetilde{z}_2] \cup [2A - 8B + 2\widetilde{z}_1, 2A + 8B + 2\widetilde{z}_1] \cup$$

$$\cup [2A - 8B + 2\widetilde{z}_2, 2A + 8B + 2\widetilde{z}_2] \cup [2A - 8B + \widetilde{z}_1 + \widetilde{z}_2, 2A + 8B + \widetilde{z}_1 + \widetilde{z}_2] \cup$$

$$\cup [A - 4B + 3\widetilde{z}_1, A + 4B + 3\widetilde{z}_1] \cup [A - 4B + 3\widetilde{z}_2, A + 4B + 3\widetilde{z}_2] \cup$$

$$\cup [A - 4B + 2\widetilde{z}_1 + \widetilde{z}_2, A + 4B + 2\widetilde{z}_1 + \widetilde{z}_2] \cup [A - 4B + \widetilde{z}_1 + 2\widetilde{z}_2, A + 4B + \widetilde{z}_1 + 2\widetilde{z}_2],$$

and discrete spectrum of the operator $\widetilde{H}_2^q$ *consists of a five eigenvalues:* $\sigma_{disc}(\widetilde{H}_2^q) = \{4\widetilde{z}_1, 4\widetilde{z}_2, 3\widetilde{z}_1 + \widetilde{z}_2, \widetilde{z}_1 + 3\widetilde{z}_2, 2\widetilde{z}_1 + 2\widetilde{z}_2,\}$ *where* $\widetilde{z}_1$, *and* $\widetilde{z}_2$, *are same concrete real number, lying the outside of the essential spectrum of the operator* $\widetilde{H}_2^q$.

d). If $-2B < \varepsilon_2 < 0$, *then the essential spectrum of the operator* $\widetilde{H}_2^q$ *consists of a single segments:*

$$\sigma_{ess}(\widetilde{H}_2^q) = [4A - 16B, 4A + 16B],$$

and discrete spectrum of the operator $\widetilde{H}_2^q$ *is empty set:* $\sigma_{disc}(\widetilde{H}_2^q) = \emptyset$.

Theorem 3. *Let* $\nu = 3$. *Then*

a). If $\varepsilon_2 = -B$ *and* $\varepsilon_1 < -6B$ *(respectively,* $\varepsilon_2 = -B$ *and* $\varepsilon_1 > 6B$ *), then the essential spectrum of the operator* $\widetilde{H}_2^q$ *consists of the union of four segments:*

$$\sigma_{ess}(\widetilde{H}_2^q) = [4A - 16B, 4A + 16B] \cup [3A - 12B + z, 3A + 12B + z] \cup$$

$$\cup [2A - 8B + 2z, 2A + 8B + 2z] \cup [A - 4B + 3z, A + 4B + 3z],$$

and discrete spectrum of the operator $\widetilde{H}_2^q$ *consists of a single eigenvalue:* $\sigma_{disc}(\widetilde{H}_2^q) = \{4z\}$, *where* $z = A + \varepsilon_1$, *lying the below (respectively, above) of the essential spectrum of the operator* $\widetilde{H}_2^q$.

b). If $\varepsilon_1 < -\frac{6B}{W}$ and $\varepsilon_2 = -2B$ or $\varepsilon_2 = 0$ (respectively, $\varepsilon_1 > \dfrac{6B}{W}$ and $\varepsilon_2 = -2B$ or $\varepsilon_2 = 0$), then the essential spectrum of the operator $\widetilde{H}_2^q$ consists of the union of four segments:

$$\sigma_{ess}(\widetilde{H}_2^q) = [4A - 16B, 4A + 16B] \cup [3A - 12B + \widetilde{z}, 3A + 12B + \widetilde{z}] \cup$$

$$\cup [2A - 8B + 2\widetilde{z}, 2A + 8B + 2\widetilde{z}] \cup [A - 4B + 3\widetilde{z}, A + 4B + 3\widetilde{z}],$$

and discrete spectrum of the operator $\widetilde{H}_2^q$ consists of a single eigenvalue: $\sigma_{disc}(\widetilde{H}_2^q) = \{4\widetilde{z}\}$, where $\widetilde{z}$, same concrete real number, and $W \approx 1,516$, lying the below (respectively, above) of the essential spectrum of the operator $\widetilde{H}_2^q$.

c). If $\varepsilon_1 = 0$ and $\varepsilon_2 > 0$ or $\varepsilon_1 = 0$ and $\varepsilon_2 < -2B$, and the condition $EW < 36B^2$ is implements, where $E = \dfrac{(B + \varepsilon_2)^2}{(\varepsilon_2^2 + 2B\varepsilon_2)}$, then the essential spectrum of the operator $\widetilde{H}_2^q$ consists of the union of ten segments:

$$\sigma_{ess}(\widetilde{H}_2^q) = [4A - 16B, 4A + 16B] \cup [3A - 12B + \widetilde{z}_1, 3A + 12B + \widetilde{z}_1] \cup$$

$$\cup [3A - 12B + \widetilde{z}_2, 3A + 12B + \widetilde{z}_2] \cup [2A - 8B + 2\widetilde{z}_1, 2A + 8B + 2\widetilde{z}_1] \cup$$

$$\cup [2A - 8B + 2\widetilde{z}_2, 2A + 8B + 2\widetilde{z}_2] \cup [2A - 8B + \widetilde{z}_1 + \widetilde{z}_2, 2A + 8B + \widetilde{z}_1 + \widetilde{z}_2] \cup$$

$$\cup [A - 4B + 3\widetilde{z}_1, A + 4B + 3\widetilde{z}_1] \cup [A - 4B + 3\widetilde{z}_2, A + 4B + 3\widetilde{z}_2] \cup$$

$$\cup [A - 4B + 2\widetilde{z}_1 + \widetilde{z}_2, A + 4B + 2\widetilde{z}_1 + \widetilde{z}_2] \cup [A - 4B + \widetilde{z}_1 + 2\widetilde{z}_2, A + 4B + \widetilde{z}_1 + 2\widetilde{z}_2],$$

and discrete spectrum of the operator $\widetilde{H}_2^q$ consists of a five eigenvalues: $\sigma_{disc}(\widetilde{H}_2^q) = \{4\widetilde{z}_1, 4\widetilde{z}_2, 3\widetilde{z}_1 + \widetilde{z}_2, \widetilde{z}_1 + 3\widetilde{z}_2, 2\widetilde{z}_1 + 2\widetilde{z}_2, \}$ where $\widetilde{z}_1$, and $\widetilde{z}_2$, are same concrete real number, lying the outside of the essential spectrum of the operator $\widetilde{H}_2^q$.

Refrences

1. Tashpulatov S. M., Parmanova R. T. *Spectra of the Energy Operator of Four-Electron Systems in the Impurity Hubbard Model. Triplet State* // Journal of Applied Mathematics and Physics. 2021. V. 9. № 11. P. 2776–2795.