UDC 624.072

ABOUT THE CRITERION OF THE APPLICABILITY OF THE RIGID CONNECTION REPLACEMENT FOR THE FLEXIBLE JOINTS IN THE CALCULATION OF A TRUSS

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The influence of the rigid connection of the rods in the truss joints on the parameters of its tense and strained state has been studied. We consider the junction scheme of the truss loading. The criterion of the relevancy of the replacement of the rigid joints of the real truss for the flexible ones has been generally formulated. The application of the formulated criterion has been illustrated by the specific example.

The real truss represents a geometrically unchangeable rod construction with the rigid connection of the rectilinear rods in the joints. The system received by the replacing of the rigid connections with the flexible joints is the design scheme of the truss, and the classical methods of the calculation of the truss formulated in the first half of the XIX century in works of Cremona, Maxwell, Ritter, Shvedlera, D. I. Zhuravsky are based on it. The classical methods are applied to the calculation of the truss made of various constructional materials - reinforced concrete, metal, wood.

The question of the relevancy of the use of the flexible joint design scheme at the strength calculation of the truss was raised for the first time at the end of the XIX century – the beginning of the XX century in the connection with the emergence of the methods of the truss calculation taking into account the rigid connection of rods in joints. The names of Mora, Manderla, Engessera, Muller-Breslau, E. O. Patton, G. P. Perederiya, N. V. Nekrasov, M. P. Danilovsky are connected with the development of these methods.

The conducted pilot studies [1, 2] confirmed a possibility of the use of the swivel design scheme when calculating trusses for the nodal scheme of loading and certain combinations of the rigid parameters of trusses. However in the same researches it has been shown that in some cases there can be essential additional normal tension which reason is a rigid connection of rods in joints of trusses.

Recently, the task of the calculation of trusses with the rigid joints has acquired relevance again in connection with the application of polymeric composite materials as constructional material. The work [3] devoted to the research of the deformation (VAT) of the tense and strained bridge fiberglass trusses points at it. The given results [3] confirm an essential divergence of the normal tension numbers when defined according to the flexible joint design scheme and taking into account the rigidity of the joints.

One of tendencies of the modern development of the construction mechanics is the enhancement of the calculation models of the constructions for the purpose of a more effective use of durability and rigidity of the real constructions. That is why there is a need of the formulation of the criterion of the applicability of the swivel design scheme when calculating the real trusses with the nodal scheme of loading.

Strictly speaking, the real trusses represent statically indefinable not free frame designs. The peculiarity of such frame designs is that they remain geometrically unchangeable at the conditional replacement of the rigid joints with the flexible joints. The acquired flexible joint - rod system can be considered as the main system of a method of forces when calculating the trusses.

The canonical equations of a method of forces represent a system of the linear nonuniform algebraic equations and generally have an appearance

$$\begin{aligned} &\delta_{11}X_1 + \ldots + \delta_{1n}X_n + \Delta_{1P} = 0, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\delta_{n1}X_1 + \ldots + \delta_{nn}X_n + \Delta_{nP} = 0. \end{aligned} \tag{1}$$

where $X_1, \ldots, X_n$ – reactions of the connections removed at formation of the main system which are the main unknown. In this case when calculating trusses by the method of forces on action of nodal loading with use of the main system as a flexible joint – rod system the main unknown will be joint points.

The coefficients included in (1) and free members are single and load movements and are determined by Maxwell-Mora's formula. When determining the movements in flat frame designs this formula looks as follows [4]

$$\Delta_i = \sum_k \int_l \frac{m_i M_P}{EI_z} ds + \sum_k \int_l \frac{n_i N_P}{EA} ds\,. \tag{2}$$

The composed elements (2) in the right part reflect the influence on the movement size, and respectively, the flexural and longitudinal deformations arising in design cores at various schemes of loading.

Taking into account (2) the structure of coefficients of the initial equations (1) for real truss designs will look like

$$\delta_{ij} = \delta_{ij}^{M} + \delta_{ij}^{N}\ , \tag{3}$$

where $\delta_{ij}^{M} = \sum_{k} \int_{l} \frac{m_i m_j}{EI_z} ds$, $\delta_{ij}^{N} = \sum_{k} \int_{l} \frac{n_i n_j}{EA} ds$ – are shares of the coefficients considering influence, respectively, flexural and longitudinal deformations in single conditions of the main system.

The free members of the initial equations will have a similar structure (1)

$$\Delta_{iP} = \Delta_{iP}^{M} + \Delta_{iP}^{N}, \tag{4}$$

where $\Delta_{iP}^{M} = \sum_{k} \int_{l} \frac{m_i M_P}{EI_z} ds$, $\Delta_{iP}^{N} = \sum_{k} \int_{l} \frac{n_i N_P}{EA} ds$ – are the shares of the free members considering influence, respectively, flexural and longitudinal deformations in a cargo condition of the main system.

As the main system of a method of forces at calculating the trusses is a pivotally - rod system with the nodal scheme of loading, the bending moments in all cores are identically equal to zero in a cargo state, and arising in them longitudinal forces, are constant on the length of each core. Therefore, the structure of free members of the initial equations for the trusses (1) changes and takes a form of

$$\Delta_{iP} = \Delta_{iP}^{N}. \tag{5}$$

In its turn, the bending moments, as well as the longitudinal forces arise in the single conditions of the main system, which are formed by the application of the single moments instead of focal points of a truss. Therefore, the structure of coefficients of the initial equations (1) doesn't change for the trusses and takes the form (3).

Therefore, if when calculating trusses the influence of the longitudinal deformations is not to be considered, then the coefficients of the initial equations (1) are equal to

$$\delta_{ij} = \delta_{ij}^{M}\ , \tag{6}$$

and all free members are identically equal to zero

$$\Delta_{iP} \equiv 0\ . \tag{7}$$

Then taking into account (6), (7) the initial equations (1) turn into the system of the linear uniform algebraic equations

$$\begin{gathered} \delta_{11}^{M} X_1 + ... + \delta_{1n}^{M} X_n = 0, \\ .... \\ \delta_{n1}^{M} X_1 + ... + \delta_{nn}^{M} X_n = 0. \end{gathered} \tag{8}$$

As the initial equations of a method of forces are linearly independent, the determinant of the system is not equal to zero, and its decisions (the focal points of a truss) have a form of

$$X_1 = 0, ..., X_n = 0\ .$$

Therefore, in case of not accounting the longitudinal deformations, the real farm with the rigid knots is equivalent to the pivotal rod system that happens to occur when replacing the rigid joints for the flexible ones.

In case of the accounting of the longitudinal deformations, the initial equations have a form

$$\begin{gathered} \left(\delta_{11}^{M} + \delta_{11}^{N}\right) X_1 + ... + \left(\delta_{1n}^{M} + \delta_{1n}^{N}\right) X_n + \Delta_{1P}^{N} = 0, \\ .... \\ \left(\delta_{n1}^{M} + \delta_{n1}^{N}\right) X_1 + ... + \left(\delta_{nn}^{M} + \delta_{nn}^{N}\right) X_n + \Delta_{nP}^{N} = 0. \end{gathered} \tag{9}$$

and in this case the focal points of a truss aren't equal to zero. The emergence of the focal points will lead to the change of the longitudinal forces in the truss cores. The longitudinal force in this case will be equal in any core of a truss

$$N_k = \sum_{i=1}^{n} n_{ki} X_i + N_{kP} .$$

The change of the internal efforts in the cores of the truss will cause the emergence of the additional normal tension, the reason of which is the rigid connection of the cores in the knots of the truss.

Thus, the consideration of the longitudinal deformations when calculating trusses with rigid knots leads to change of parameters of the intense deformed state (VAT). Therefore, criterion of legitimacy of replacement of rigid knots of real trusses at their calculation on action of nodal loadings hinged is the possibility of not accounting of longitudinal deformations of cores of the truss when determining its VAT parameters.

We will carry out the numerical assessment of the influence of the longitudinal deformations on the VAT parameters of a truss on a private example. Let's consider a two-rod truss with the rigid connection in the knot under the influence of any nodal loading (Fig. 1)

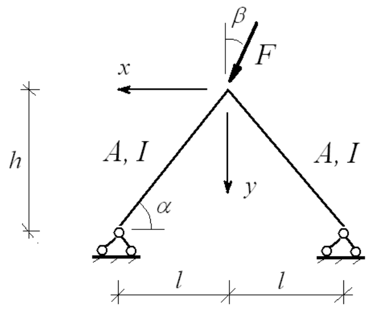

Fig. 1. The two-rod truss

The constructional material is considered uniform isotropic linearly elastic body which behavior is described by the module of elasticity of E identical at stretching and compression.

The calculation of the truss is performed by a method of forces. As the main system, the truss with hinged connection of cores in the knot undertakes (Fig. 2)

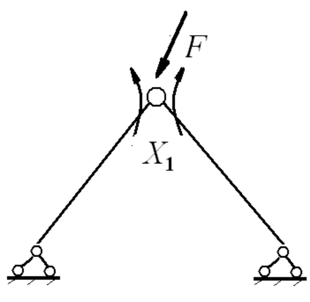

Fig. 2. The main system of a method of forces

The initial equation of a method of forces shows as

$$\delta_{11} X_1 + \Delta_{1P} = 0 . \tag{10}$$

The main unknown in the equation (10) is the bending moment arising in the knot owing to the rigid connection of the cores of the truss.

The entering (10) coefficient and the free member are calculated on Maxwell-Mora's formula which takes into account the influence of the flexural and longitudinal deformations as shown

$$\Delta_i = \sum_k \int_l \frac{m_i M}{EI_z} ds + \sum_k \int_l \frac{n_i N}{EA} ds .$$

The internal efforts of a single state connected with the calculation of the coefficient and the free member in a dimensionless form are shown as:

– the bending moments

$$m_1(x,l) = (1-\frac{x}{l}); \;\; m_2(x,l) = (1-\frac{x}{l});$$

– longitudinal forces

$$n'_{11}(\alpha) = \cos\alpha; \;\; n'_{21}(\alpha) = \cos\alpha,$$

where $n'_{k1} = n_{k1}h \; (k = 1, 2)$.

The longitudinal forces of a cargo state connected with calculation of the free member in a dimensionless form can be shown as

$$N'_{1P}(\alpha,\beta) = -0.5\frac{\cos\beta}{\sin\alpha}(1+\frac{\operatorname{tg}\beta}{\operatorname{tg}\alpha}); \;\; N'_{2P}(\alpha,\beta) = 0.5\frac{\cos\beta}{\sin\alpha}(\frac{\operatorname{tg}\beta}{\operatorname{tg}\alpha}-1),$$

where $N'_{1P} = \frac{N_{1P}}{F}$, $N'_{2P} = \frac{N_{2P}}{F}$.

The bending moments in both cores are identically equal in a cargo state to zero.

Taking into account the calculation of the coefficient and the free member, the bending moment arising in the rigid knot of a truss in a dimensionless form is shown as

$$X'_1(\alpha,\beta,\lambda) = 1.5\frac{\cos\beta}{\cos^2\alpha(3+\lambda^2\operatorname{tg}^2\alpha)},$$

where $X'_1 = \frac{X_1}{Fl}$;

$\lambda = \frac{l_c}{i}$ – the parameter of the flexibility of a core of a truss depending on its length l_c and radius of inertia of cross section i.

The dependence of the size of the bending point received focal on truss parameters α, β, λ it is presented on schedules Figure 3.

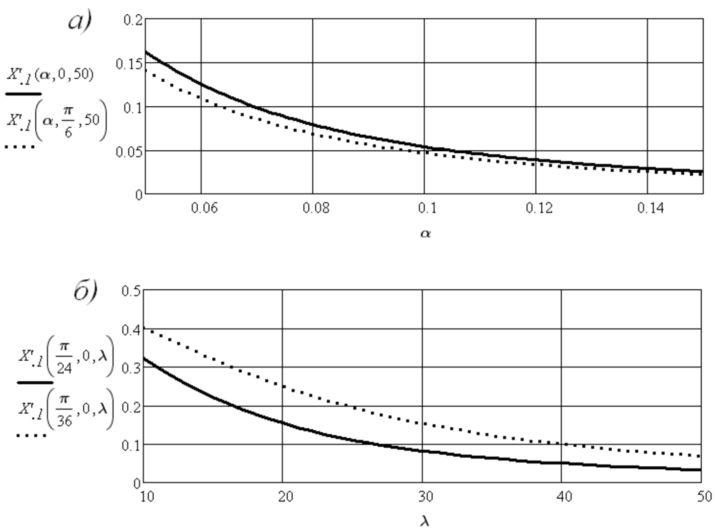

Fig. 3. The schedules of the dependence of the size of the focal bending point on the truss parameters

As follows from the schedules submitted in Figure 3, *a* the size of the focal bending point increases with reduction of a corner α and decreases with increase in a corner β, that is at emergence in the knot of a horizontal component of nodal loading. In a limit when values $\alpha, \beta \rightarrow 0$, the focal bending point comes to the value of the frame the bending moment $\frac{Fl}{2}$ arising in a beam with the size of flight and the scheme of loading identical with the considered truss. As it follows from the schedules submitted in Figure 3, *b* the size of the focal bending point increases with the reduction of the flexibility of the cores of a truss.

The dimensionless increments of longitudinal forces in the cores of a truss arising owing to emergence of the focal bending point are described by formulas

$$\Delta N_1(\alpha,\beta,\lambda) = 1 - k_1(\alpha,\beta,\lambda);$$

$$\Delta N_2(\alpha,\beta,\lambda) = 1 - k_2(\alpha,\beta,\lambda),$$

where $k_1(\alpha,\beta,\lambda) = 1 - 1.5\frac{\mathrm{tg}\,\alpha}{(3+\lambda^2\,\mathrm{tg}^2\,\alpha)(\mathrm{tg}\,\beta - \mathrm{tg}\,\alpha)}$;

$$k_2(\alpha,\beta,\lambda) = 1 + 1.5\frac{\mathrm{tg}\,\alpha}{(3+\lambda^2\,\mathrm{tg}^2\,\alpha)(\mathrm{tg}\,\beta - \mathrm{tg}\,\alpha)}$$

the coefficients of the influence of the rigidity of a knot on the longitudinal forces in the truss cores.

The dependence of the increments of the longitudinal forces on the parameters of the truss α, β, λ is presented on the schedules of Figure 4.

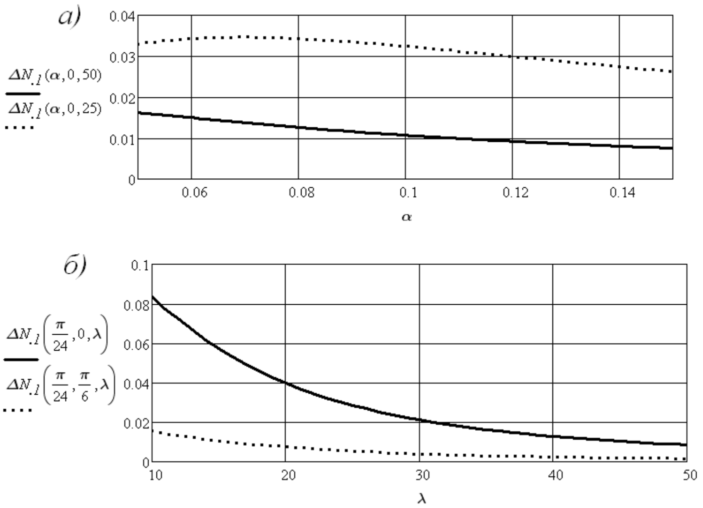

Fig. 4. The schedules of the dependence of the size of the increment of longitudinal forces on the truss parameters

As it follows from the schedules submitted in pic. 4 the size of the increments of the longitudinal forces increases with reduction of a corner α, flexibility of cores of a truss λ and decreases at emergence in knot of a horizontal component of nodal loading. At the most adverse combination of parameters of a truss the size of an increment of longitudinal forces doesn't exceed 10% of values of the longitudinal forces according to the hinged scheme of the truss.

The dimensionless increments of the normal tension in the cores of a truss arising due to the emergence of the focal bending point and the changes of the longitudinal forces as described by the formulas

$$\Delta\sigma_1(\alpha,\beta,\lambda) = 1 - k_{1\sigma}(\alpha,\beta,\lambda);$$

$$\Delta\sigma_2(\alpha,\beta,\lambda) = 1 - k_{2\sigma}(\alpha,\beta,\lambda),$$

where $k_{1\sigma}(\alpha,\beta,\lambda) = k_1(\alpha,\beta,\lambda) + \lambda\cos\alpha\frac{X_1'(\alpha,\beta,\lambda)}{N_{1P}'(\alpha,\beta)}$;

$$k_{2\sigma}(\alpha,\beta,\lambda) = k_2(\alpha,\beta,\lambda) + \lambda\cos\alpha\frac{X_1'(\alpha,\beta,\lambda)}{N_{2P}'(\alpha,\beta)}$$

coefficients of the influence of the rigidity of a knot on the normal tension in the truss cores. The dependence of the size of the increments of normal tension on parameters of the truss α, β, λ is presented on schedules (Fig. 5).

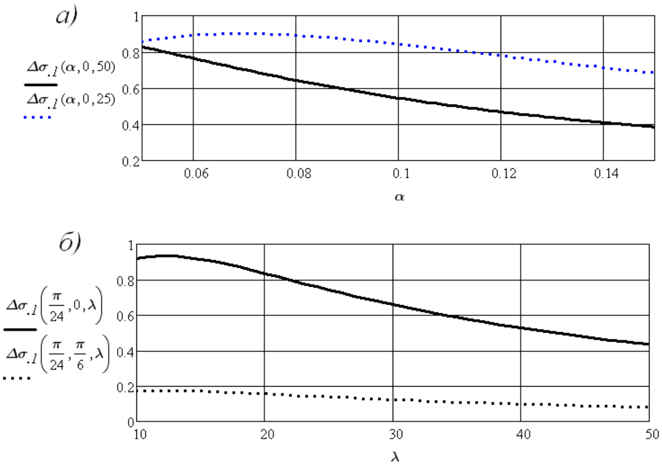

Fig. 5. The schedules of the dependence of the size of an increment
of normal tension on the truss parameters

As it follows from the submitted schedules, the size of the increment of normal tension increases with the reduction of a corner α, flexibility of cores λ and decreases at emergence in the knot of a horizontal component of nodal loading. In general it is possible to draw a conclusion at values $\alpha < \frac{\pi}{6}$ and $\lambda < 100$ the increment of normal tension makes more than 5% of normal tension in the truss cores according to the hinged scheme and at certain combinations and can reach the commensurable values.

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