

## COMPARISON OF HARMONY SEARCH METHOD AND PARTICLE SWARM OPTIMIZATION METHOD ALGORITHMS OF THE DESIGN OPTIMIZATION OF CELLULAR BEAMS

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*In the following research the problem of design optimization for the minimization of the cellular beam weight is solved using the optimum structural design algorithms that are based on stochastic search techniques which are quite effective in finding this solution to discrete programming problems.*

The design of a cellular beam requires the choice of an original rolled beam from which the cellular beam is to be produced, circular opening diameter and the spacing between the centers of these circular holes or the total number of holes in the beam. Hence, the sequence number of the rolled beam sections in the standard steel section tables, hole diameter and the total number of holes are taken as design variables in the optimum design problem considered.

In the research, the solution of the discrete non-composite cellular beam design problem given above is investigated using two meta-heuristic search techniques; namely harmony search algorithm and particle swarm optimizer.

The basic components of the harmony search algorithm can now be outlined in five steps as follows.

*Step 1. Initialization of a Parameter Set:* A harmony search optimization parameter sets are initialized first. These parameters consist of four entities called a harmony memory size (*hms*), a harmony memory considering rate (*hmcr*), a pitch adjusting rate (*par*) and a maximum search number ( $N_{cyc}$ ). It is worthwhile to mention that in the standard harmony search algorithm these parameters are treated as static quantities, suitable values are chosen within their recommended ranges of  $hmcr \in (0.70 \sim 0.95)$  and  $par \in (0.20 \sim 0.50)$ . It should be mentioned that the selection of these values is the problem dependent and it requires number of trials to identify the appropriate ones.

*Step 2. Initialization and Evaluation of Harmony Memory Matrix:* A harmony memory matrix *H* is generated and randomly initialized next. This matrix incorporates (*hms*) number of feasible solutions. Each solution (harmony vector,  $I^j$ ) consists of *nv* integer numbers between 1 to *ns* selected randomly each of which corresponds to sequence number of design variables in the design pool, and is represented in a separate row of the matrix; consequently the size of *H* is (*hms*  $\times$  *nv*).

$$H = \begin{bmatrix} I_1^1 & I_2^1 & \dots & I_{nv}^1 \\ I_1^2 & I_2^2 & \dots & I_{nv}^2 \\ \dots & \dots & \dots & \dots \\ I_1^{hms} & I_2^{hms} & \dots & I_{nv}^{hms} \end{bmatrix} \begin{matrix} f(I^1) \\ f(I^2) \\ \dots \\ f(I^{hms}) \end{matrix} \quad (1)$$

$I_i^j$  is the sequence number of the  $i^{\text{th}}$  design variable in the  $j^{\text{th}}$  randomly selected feasible solution. (*hms*) solutions shown in Eqn. 1 are then analyzed, and their objective function values are calculated.

*Step 3. Generating a New Harmony:* A new harmony solution vector  $I' = [I_1', I_2', \dots, I_{nv}']$  is improvised by selecting each design variable from either harmony memory or the entire discrete set. The probability that a design variable is selected from the harmony memory is controlled by a parameter called harmony memory considering rate (*hmcr*). To execute this probability, a random number  $r_i$  is generated between 0 and 1 for each variable  $I_i$ . If  $r_i$  is smaller than or equal to *hmcr*, the variable is chosen from harmony memory in which case it is assigned any value from the *i*-th column of the *H*, representing the value set of variable in *hms* solutions of the matrix Eqn. 2. Otherwise (if  $r_i > hmcr$ ), a random value is assigned to the variable from the entire discrete set.

$$I_i' = \begin{cases} I_i' \in \{I_i^1, I_i^2, \dots, I_i^{hms}\} & \text{if } r_i \leq hmcr \\ I_i' \in \{1, \dots, ns\} & \text{if } r_i > hmcr \end{cases} \quad (2)$$

*Step 4. Update of Harmony Matrix.*

*Step 5. Termination.*

The basic steps of the particle swarm optimization for a general discrete optimization problem can be outlined as follows:

Step 1. Swarm of particles is initialized randomly with sequence numbers  $I_0^i$  which corresponds positions  $I_0^i$  and initial velocities  $v_0^i$  that are randomly distributed throughout the design space. Here  $I_0^i$  represents the sequence number of values in the discrete set. These are obtained from the following expressions.

$$I_0^i = INT [I_{min} + r(I_{max} - I_{min})] \quad (3)$$

$$v_0^i = [I_{min} + r(I_{max} - I_{min}) / \Delta t] \quad (4)$$

Step 2. The objective function values  $f(x_k^i)$  are evaluated using the design space positions  $x_k^i$ .

Step 3. The optimum particle position  $p_k^i$  at the current iteration  $k$  and the global optimum particle position  $p_k^g$  are updated by equating  $p_k^i$  to  $f(x_k^i)$  and  $p_k^g$  to the best  $f(x_k^i)$ .

Step 4. The velocity vector of each particle is updated considering the particle's current velocity and position, the particle's best position and global best position, as follows:

$$v_{k+1}^i = wv_k^i + c_1r_1 \frac{(p_k^i - x_k^i)}{\Delta t} + c_2r_2 \frac{(p_k^g - x_k^i)}{\Delta t} \quad (5)$$

where  $r_1$  and  $r_2$  are random numbers between 0 and 1,  $p_k^i$  is the best position found by particle  $i$  so far, and  $p_k^g$  is the best position in the swarm at time  $k$ .  $w$  is the inertia of the particle which controls the exploration properties of the algorithm.  $c_1$  and  $c_2$  are trust parameters that indicate how much confidence the particle has in itself and in the swarm respectively.

Step 5. The sequence number for the position of each particle is updated from

$$I_{k+1}^i = INT (I_k^i + v_{k+1}^i \Delta t) \quad (6)$$

Where  $I_{k+1}^i$  is the sequence number in the discrete set for  $x_{k+1}^i$  which is the position of particle  $i$  at iteration  $k + 1$ ,  $v_{k+1}^i$  is the corresponding velocity vector and  $\Delta t$  is the time step value.

Step 6. Steps 2–5 are repeated until pre-determined maximum number of cycles is reached.

Harmony search method and particle swarm based optimum design algorithm presented above are used to design a cellular beam to compare which method is better while finding the optimum solution. For the hole diameters discrete set that has 421 values ranging from 180 mm to 600 mm with the increment of 1 mm is prepared. Another discrete set is arranged for the number of holes that contains numbers ranging from 2 to 40 with the increment of 1.

A simply supported beam shown in Figure 1 is selected as first design example to demonstrate the steps of optimum design algorithms developed for cellular beams that are based on harmony search and particle swarm methods. The beam has a span of 4 m and is subjected to 5 kN/m dead load including its own weight. A concentrated live load of 50 kN also acts at mid-span of the beam as shown in the same figure.

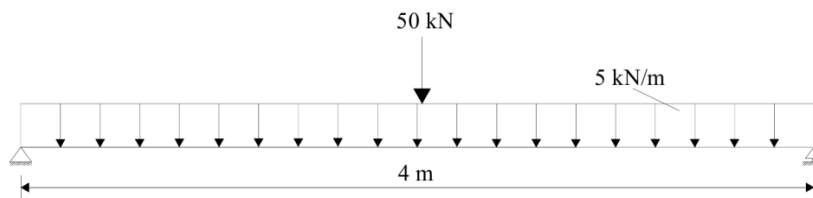


Fig. 1. Loading of 4-m simply supported beam

The maximum displacement of the beam under these point and distributed loads is restricted to 12 mm while other design constraints are implemented from BS5950. The modulus of elasticity is taken as 205 kN/mm<sup>2</sup> and Grade 50 steel is adopted for the beam which has the design strength of 355 MPa.

In the use of the harmony search method, the other parameters hmcr and par are taken as 0.8 and 0.35 respectively. It took 17 cycles for the harmony search method to fill the harmony memory matrix. This matrix is given in Table 1.

Table 1 – The initial feasible designs selected by PSO and HS algorithm

PSO Algorithm (Initial)					HS Algorithm (After 17 iterations)				
Particle No	Section (UB)	Number ofHoles	HoleDia. (mm)	Weight (kg)	Particle No	Section (UB)	Number ofHoles	HoleDia. (mm)	Weight (kg)
1	356×127×33	8	386	120,53	1	356×127×39	10	316	144,8
2	406×140×39	8	429	137,71	2	406×178×60	6	412	231,48
3	406×140×39	7	366	148,2	3	406×178×74	7	538	255,95
4	305×102×33	8	342	124,06	4	406×178×74	7	396	284,5
5	406×178×74	8	380	280,79	5	457×191×74	7	352	287,56
6	356×171×57	10	291	217,6	6	457×191×98	5	581	371,18
7	305×127×37	9	371	132,54	7	533×210×109	6	558	398,71
8	356×171×45	8	324	172,2	8	533×210×122	5	560	464,99
9	406×178×67	8	365	255,68	9	610×229×125	6	499	476,21
10	305×127×37	8	349	139,32	10	686×254×140	4	590	543,51

The new objective function value 163.44 kg is better than the worst harmony in the memory matrix 543.51 kg. Hence this new design is placed in the 2th row of the harmony memory matrix and the worst design with the largest objective function value is discarded from the harmony memory matrix. The new design does not affect the first row of the harmony memory matrix in this search. However later, when the harmony search algorithm continues to seek better designs another vector that is obtained in later cycles changes the harmony memory matrix.

Table 2 – Feasible designs obtained after 780 iterations by PSO and HS algorithms

PSO Algorithm (Initial)					HS Algorithm (After 17 iterations)				
Particle No	Section (UB)	Number ofHoles	HoleDia. (mm)	Weight (kg)	Particle No	Section (UB)	Number ofHoles	HoleDia. (mm)	Weight (kg)
1	305×102×25	9	374	86,27	1	305×102×25	9	402	82,19
2	305×102×25	10	365	83,19	2	305×102×25	10	345	82,5
3	254×146×4	10	336	157,78	3	305×102×25	10	342	82,8
4	305×102×25	9	351	89,1	4	305×102×25	10	315	82,9
5	305×102×25	10	348	85,81	5	305×102×25	10	312	83,11
6	305×102×25	11	334	84,15	6	305×102×25	9	335	83,52
7	305×102×25	11	330	84,8	7	305×102×25	10	305	83,72
8	254×146×37	12	303	133,65	8	305×102×25	9	328	83,82
9	305×102×25	11	333	84,32	9	305×102×25	9	313	84,33
10	305×102×25	10	368	82,71	10	305×102×25	9	310	84,43

The optimum result presented in Table 2 is obtained after 780 iterations. It is noticed that this design vector remained the same even though the design cycles are continued to reach 5000 which was the pre-selected maximum number of iterations.

Table 3 – Comparison of optimum designs for 4-m simply supported beam

Search Method	OptimumSection Designations (UB)	Diameterof Hole (mm)	TotalNumber ofHoles	Minimum Weight(kg)
HS Algorithm	305×102×25	402	9	82,19
PSO Algorithm	305×102×25	368	10	82,71

It is apparent from the Table 3 that the optimum design has the minimum weight of 82.19 kg. In the optimum design the harmony search algorithm selects 305×102×25 UB section for the root beam. Furthermore it decides that the cellular beam should have 9 circular holes each having 402mm diameter.

Similar to harmony search algorithm the particle swarm optimizer also starts initializing the parameters. The values  $c_1$  and  $c_2$  are selected as 1, 2 is adopted for  $w$  and the values of  $\Delta t$  and  $V_{max}$  are chosen as 2. The total number of particles is selected as 10. The initial set of feasible designs assigned to each particle is listed in Table 1. In this table, the first particle has the feasible design with minimum weight. This design has the minimum weight of 120.53 kg where the universal beam section of 356×127×33 UB is selected for the root beam. The beam should be produced such that it should have 8 circular holes each having 386mm diameter at this weight. Design cycles are started with these values of the particles and the positions and the objective function values of particles keep on changing iteration after iterations. The best among these positions is kept as the optimum design attained in the current iteration. If this one is better than the global one then it is assumed as the optimum design obtained up to the present iteration. Table 2 contains the designs obtained after 430 iterations. It is noticed that the optimum design obtained in this table does not improve even though the iterations are continued until 5000. Comparing to harmony search method, the optimum design is obtained after 430 iterations in particle swarm algorithm. It is apparent from Table 3 that the optimum design has the minimum weight of 82.71 kg which selects 305×102×25 UB section for the root beam, total of 10 holes in the beam each having 368mm diameter. In addition, the design history curve for both techniques is shown in Figure 2.

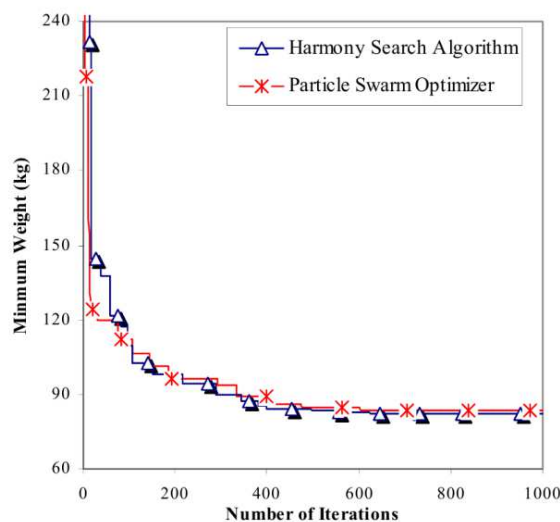


Fig. 2. The design history graph for 4-m cellular beam

**Conclusions.** In this particular problem, these results demonstrate that while both the strength and geometric constraints are dominant in HS algorithm, in PSO algorithm, only the strength constraints are severe. To sum up, the optimum result of harmony search technique is compared with particle swarm optimization method to show accuracy and performance of methods on cellular beams. Although the algorithms of HS and PSO are mathematically quite simple, they are quite robust in finding the solutions of combinatorial optimum design problems as it is demonstrated in the example considered. This result also demonstrates that harmony search algorithm is a very rapid and effective method for optimum design of small-scale problems that consist of a small number of decision variables. Consequently, the technique is recommended for its application to optimization of the three different cellular beam problems.

#### REFERENCES

1. Руководство по проектированию балок с перфорированной стенкой : утв. ЦНИИПСК 19.12.78. – М., 1978. – С. 28.
2. Steelwork design Guide to BS 5950 : Part 1, “Section Properties, Member Capacities”, Vol. 1, – Steel Construction Institute. – The 4th ed. – U.K., 1990.
3. Perez, R.E. Particle Swarm Approach for Structural Design Optimization / R.E. Perez, K. Behdinan // Computers and Structures, 85 (19-20), 1579-1588, 2007.
4. Saka, M.P. Optimum Design of Grillage Systems Using Harmony Search Algorithm / M.P. Saka // Journal of Structural and Multidisciplinary Optimization. – 2009. – Vol. 38 (1). – P. 25–41.