UDC 624.078.4

RESISTANCE OF THE PRECAST PRESTRESSED HOLLOW-CORE SLABS WITH ADDITIONAL REINFORCEMENT OVER SUPPORT IN PLATFORM JOINT

ANDREI LAZOUSKI
Polotsk State University, Belarus

VIKTOR TUR
Brest State Technical University, Belarus

Analytical model allowing calculation of relation "bending moment - rotation angle" based on general deformation model and block model of reinforced concrete resistance and describing stress-strain state of the precast prestressed hollow-core slabs in platform joint zone is considered. The model considers transversal pressure, ratio of reinforcement installed in concrete of hollow void filling to concrete, achievement of yield strength of reinforcement.

The proposed analytical model for the stress-strain state assessment of the precast prestressed hollow-core floor slabs in platform joint zone is based on block resistance model for bending reinforced concrete element, initially offered by H. M. Westergaard in 1930 [1]. This block model was developed by P.I. Vasilyev, E.N. Peresypkin [2], V.I. Belov [3], S.E. Peresypkin [4], Y.V. Pochinok [5], M.V. Brovkina [6], P. Croce, P. Formichi [7], A. Borosnyói, and G.L. Balázs [8], A. Casanova, L. Jason and L. Davenne [9] etc. The basic assumptions of the analytical model are stated in previous works [10].

The maximum value of the bending moment from concentrated load P_1, which is applied at L distance from the axis of platform joint (support) is given by:

$$M = P \cdot (L - l); \tag{1}$$

At the same time bending moment in the plastic hinge formation zone does not extend to the end sections of slabs, which are cutting off by critical cracks and influencing vertical pressure (Fig. 1).

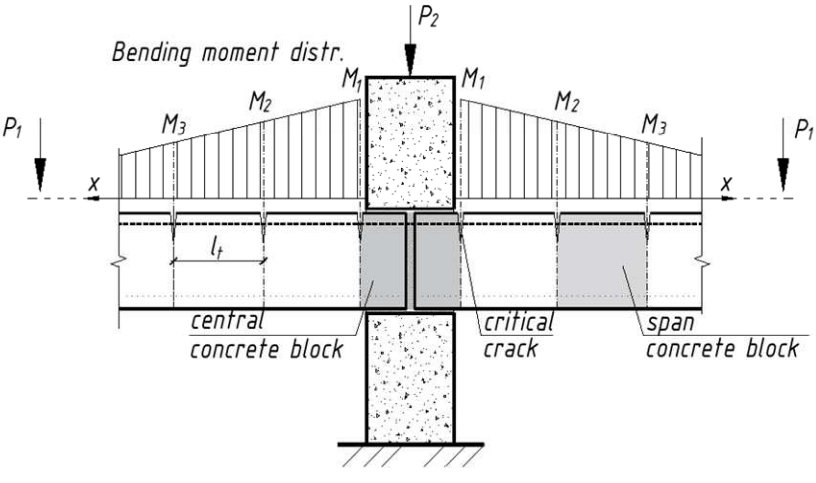

Fig. 1. The scheme of slabs dividing into blocks between cracks

Concrete blockstaken for the analysis and allocated by neighboring cracks along the element length include: central block between two critical cracks with vertical grout in platform joint zone, span concrete blocks to the left and to the right of platform joint with free ends. In case of geometrically and physically symmetric system it is possible to consider the span block on one side of platform joint only.

Crack width between two concrete blocks is defined as the sum of slip $s(x)$ between reinforcement and tension concrete along the transmission zones l_t to the left (l) and to the right (r) from crack edges:

$$w = \int_{-l_{t(l)}}^{l_{t(r)}} s(x)dx = \int_{-l_{t(l)}}^{l_{t(r)}} [\varepsilon_s(x) - \varepsilon_{ct}(x)]dx. \tag{2}$$

The crack is formed on the end area of transmission zone where the strain in reinforcement is the same as that in the surrounding concrete ($\varepsilon_s = \varepsilon_{ct}$), under bending moment, which is equal to the cracking moment M_{crc}. At the subsequent loading stages, geometrical and physical properties of slabs and of hollow voids filling along their span concrete blocks are allocated with length of: $L_m < [l_{t(l)} + l_{t(r)}]$ or $L_m \geq [l_{t(l)} + l_{t(r)}]$, depending on moment distribution.

In the first case (Fig. 2, *a*), on the whole length of concrete block strain of concrete at the reinforcing bars axis does not exceed the ultimate tensile concrete strain $\varepsilon_{(ct)1}$, i.e. transmission zones on the left (*l*) and on the right (*r*) are overlapped, and the condition $\varepsilon_{ct} < \varepsilon_{(ct)1}$ is valid for any cross-section in concrete block. For such concrete block the first stage of crack formation is completed and at the subsequent loading stages there is a re-distribution of stresses between reinforcement and tensile concrete without formation of new cracks in the form of accumulation of slip which can be seen in the opening of cracks.

In the second case (Fig. 2, *b*), in the middle part of concrete block length there is a joint deformation zone of reinforcement and concrete with length of $[L_m - (l_{t(l)} + l_{t(r)})] > 0$, i.e. transmission zones on the left (*l*) and on the right *(r)* are not overlapped. The concrete block at this stage of loading is at the first stage of crack formation. At the subsequent stages of loading depending on moment distribution from external loading, geometrical and physical properties of slabs and of hollow voids filling along their span for the considered concrete block there are two possible ways: further division by new cracks on smaller concrete blocks (as it is described above), or its transition to the second (established) crack formation stage.

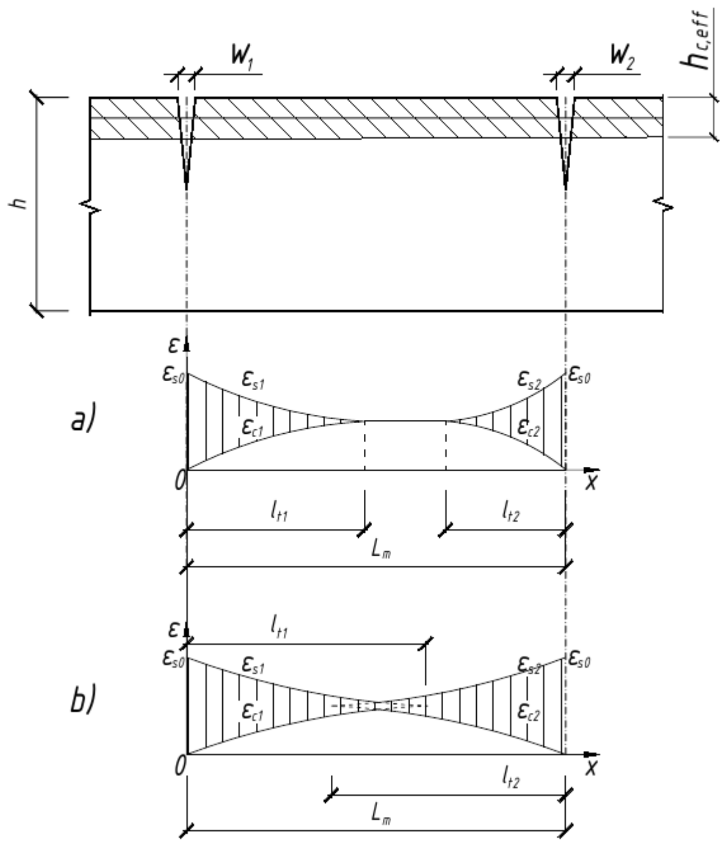

Fig. 2. Initial stage of crack formation (1st case) (*a*) and redistribution of stresses between reinforcement and tensile concrete (2nd case) (*b*)

Central concrete block with cracks on both sides of the joint and vertical grout directly in zone of platform joint are further considered. As it was pointed, the central concrete block in case of pressing by vertical bearing walls is under the transversal pressure influencing the bond of additional and slab's reinforcement and with the surrounding concrete.

For the assessment of reinforcement and tensile concrete strains distribution along the central concrete block length, subjected to transversal pressure, on the basis of taken assumptions and the bond-slip law $\tau_b = f(s)$ effective area of concrete of tensile zone $A_{c,eff}$ with length of dx can be taken with a bar of additional reinforcement over support with area of $A_{s,ad}$ (Fig. 3).

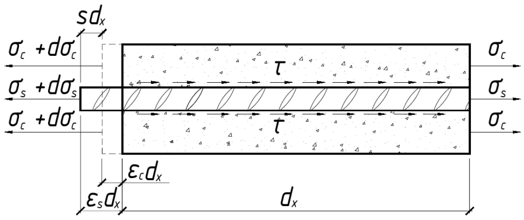

Fig. 3. Free body equilibrium for the central concrete block [7]

Tension of the upper prestressed reinforcement of slabs at their end faces as a part of surrounding concrete of central block tension zone is considered in the specified cross sectional area, $A_{c,red} = A_{c,eff} + \alpha_E \cdot A_p$. According to the accepted assumptions bending moment from external loading along the central block influenced by vertical pressing is absent. The *N* force in the tension zone of the central concrete block from action of tensile stresses in cross-sections with cracks on the ends of the block without taking into account tension of the vertical elements transferring pressure at geometrically and physically symmetric system is distributed uniformly along the block (otherwise, linear distribution of *N* force along of the block is considered).

For any cross-section at the block length, the total force in tensile concrete N_c and reinforcing bar N_s is constant on the whole length of the block. Then equilibrium conditions within concrete block should be written:

$$d\sigma_s \cdot A_{s.ad} + d\sigma_{ct} \cdot A_{c,red} = 0\,. \tag{3}$$

From equilibrium conditions the stresses in reinforcing bar with the diameter of Ø *as a resultant of the* tensile stresses $A_{s,ad}$ are distributed on its cross sectional area, and shear (tangent) stresses τ_b on the contact area of reinforcing bar with surrounding concrete, and also differences of the strains of reinforcement ε_s and concrete ε_{ct} as the slip *s*, the bond-slip law $\tau_b = f(s)$, connecting tangent stress on the contact area of the reinforcing bar with concrete and its slip considering the transversal pressure can be written:

$$d\sigma_s \cdot \frac{\pi\varnothing^2}{4} = \pi\varnothing \cdot \Omega_{p,tr} \cdot \tau_b(s)dx \tag{4}$$

$$(\varepsilon_s - \varepsilon_c)dx = ds \tag{5}$$

$$\Omega_{p,tr} \cdot \tau_b = f(s) \tag{6}$$

The parameter $\Omega_{p,tr}$ for bond-slip law takes into account transveral pressure p_{tr} in concrete of average strength f_{cm} according to [11].

If reinforcement strains achives and exeeds the yield strains $\varepsilon_s \geq \varepsilon_{sy}$ QUOTE (s? ≥ sy?) the reducing parameter Ω_y is taken for the bond-slip law [11].

For the resistance problem solution, the central concrete block in the tension zone of section depth h_{eff}, allocated with the neighboring cracks, is divided with *n* cross-sections into (*n* – 1) intervals with length of Δx .The interval Δx_k borders are matched with borders of vertical grout between end faces of hollow-core slabs (see Fig. 2). At the same time, taking into account a low adhesion of the concrete of vertical grout filling with concrete of the slabs, transversal tension of the concrete of the grout in the sections matching its borders is not considered. Therefore on the interval equal to length of vertical grout Δx_k, it is accepted the "σ-ε" diagram of for concrete of hollow voids filling and that the effective cross sectional area of the tension zone in concrete of filling $A_{c,eff,ad}$ without consideration of upper reinforcement of slabs.

Equations (4)..(6) can be rewritten as a following system of equations:

$$\begin{cases} \dfrac{d}{dx}s = \varepsilon_s(\sigma_s) - \varepsilon_{ct}\left(\dfrac{N - \sigma_s A_s}{A_{c,eff}}\right); \\ \dfrac{d}{dx}\sigma_s = \dfrac{4}{\varnothing}\Omega_y \Omega_{p,tr}\tau(s). \end{cases} \tag{7}$$

Since boundary conditions depend on the strain distribution in reinforced concrete element, the solution of system (7) requires an iterative procedure. The proposed procedure is based on the iterative solution of system of the differential equations (7). Also for the central concrete block the process of crack formation takes place into two stages as well as for span concrete blocks. The feature of the central block is the reduction area of concrete in the tension zone in the place of vertical grout (only concrete of hollow voids filling $A_{c,eff}$ is taken into consideration) that during symmetric loading of hollow-core slabs to the left and to the right from platform joint at the end of the first stage of crack formation facilitates the appearing of crack in this place.

For cracked sections (at the edges of the allocated block) values of streses $\sigma_{s,(l)}$ and $\sigma_{s,(r)}$ are known, but values of slips $s_{(l)}$ and $s_{(r)}$ – are unknown. The iterative procedure of the solution of the system of equations (7) for the central block is as follows:

1. Initial parameters of iterative process are assumed: the block length (ΔL), the area of tensile reinforcement (A_s), the effective area of the tensile concrete $A_{c,eff}$, the N force corresponding to force in the tension zone of the considered central block. In the first approximation the initial slip s_0 is recommended to take equal to product of the maximum reinforcement strain $\varepsilon_{s,max}$ at the cracked section with and block length (ΔL):

$$s_0 = \varepsilon_{s,\max} \cdot \Delta L \,. \tag{8}$$

2. Diagrams "σ-ε" for reinforcing steel and concrete, bond-slip law "τ-*s*" are defined. Besides, the parameters Ω_y and $\Omega_{p,tr}$ [11] are taken into account if the effects of yield strength and transversal pressure of concrete block are considered.
3. The considered central block is divided into (n) elements of equal length Δx.
4. As the origin of axis x (x = 0) cracked section is taken. Stress in reinforcement ($\sigma_{s,0}$) and in tension concrete are known ($\sigma_{ct,0} = 0$). Under the taken bond-slip law tangent stress $\tau_{b,0}(s_0)$ is calculated.
5. Tension stress in reinforcing bar at *the first* (n = 1) section at distance Δx from origin of axis x is calculated:

$$\sigma_{s,1} = \sigma_{s,0} - \frac{4}{\varnothing} \cdot \tau_{b,0} \cdot \Delta x. \tag{9}$$

6. Concrete tension stress at the considered section is calculated:

$$\sigma_{ct,1} = \frac{\left(N - \sigma_{s,1} \cdot A_s\right)}{A_{c,eff}}. \tag{10}$$

7. Reinforcing bar $\varepsilon_{s,1}$ and concrete $\varepsilon_{ct,1}$ strains are defined from "σ-ε" diagrams.
8. The slip s is calculated for the considered section (n = 1):

$$s_1 = s_0 - (\varepsilon_{s,1} - \varepsilon_{c,1})\Delta x \tag{11}$$

9. Under the taken bond-slip law tangent stress $\tau_{b,1}(s_1)$ is defined.
10. The iteration procedure repeats for the subsequent sections. For the any *i*- section at the length of the concrete block the equations can be written as:

$$\sigma_{s,i} = \sigma_{s,i-1} - \frac{4}{\varnothing} \cdot \tau_{b,i-1} \cdot \Delta x \,, \tag{12}$$

$$\sigma_{ct,i} = \frac{\left(N - \sigma_{s,i} \cdot A_s\right)}{A_{c,eff}} \,, \tag{13}$$

$$s_i = s_{i-1} - (\varepsilon_{s,i} - \varepsilon_{c,i})\Delta x \,. \tag{14}$$

Criteria of convergence of the iterative procedure are the achievement of concrete strain the values of the ultimate strain of concrete ($\varepsilon_{ct} = \varepsilon_{ctu,fl}$) that indicates a new crack formation; achievement of concrete strain the strain of reinforcement ($\varepsilon_{ct} = \varepsilon_s$) that indicates the completion of zone of stresses transition and the beginning of zone of joint deformations of concrete and reinforcement. As soon as the convergence is reached, the iterative procedure is finished. Otherwise, new value of initial parameters are defined and the iterative process repeats.

After the achievement of the iterative procedure convergence, the distributions of strains along the concrete block $\varepsilon_s(x)$ and $\varepsilon_{ct}(x)$ and the location of neutral axis in sections with crack are received. In this case distribution of section curvatures along the length of the block allocated with cracks is possible to calculate:

$$\varphi_i = \frac{\varepsilon_{s,i}}{d - x_{c,i}} = 0\,, \qquad (15)$$

where d – effective depth of section;

$x_{c,i}$ – depth of the neutral axis in i section. For intermediate sections of concrete block is calculated by linear interpolation of change between x_1 and x_n – values on the ends of the block.

For the known curvatures φ_i distribution along the block length, rotation angle are determined by integration of the equation:

$$\theta = \int_{l_p} \varphi(x)dx\,, \qquad (16)$$

where $\varphi(x)$ – distribution function of curvatures along the block L for any stage of loading.

Carrying out monotonous loading the relation "bending moment – rotation angle" is received which is applied in nonlinear analysis of structural system.

Conclusions. The proposed analytical model is based on general deformation model and block model of reinforced concrete resistance and describe stress-strain state of precast prestressed hollow-core slabs in platform joint zone. The model considers transversal pressure, ratio of reinforcement installed in concrete of hollow void filling to concrete, achievement of yield strength of reinforcement. The model allows calculating relation "bending moment on the support – rotation angle" for nonlinear deformation of hollow-core floor slabs in the zone of platform joints of buildings in nonlinear analytical methods.

REFERENCES

1. Westergaard, H.M. Computation of Stresses in Bridge Slabs Due to Wheel Loads / H.M. Westergaard // Public Roads. – 1930. – Vol. 11, No. 1. – P. 1–23.
2. Васильев, П.И. Раскрытие швов и трещин в массивных бетонных конструкциях / П.И. Васильев, Е.Н. Пересыпкин // Аннотации законченных в 1967 г. научно-исследовательских работ по гидротехнике. – Л. : Энергия, 1968.
3. Белов, В.И. К вопросу исследования напряжённо-деформированного состояния железобетонных балок как систем, составленных из упругих блоков / В.И. Белов, П.И. Васильев, Е.Н. Пересыпкин // Вопросы прочности бетона и железобетонных конструкций : тр. координационных совещаний по гидротехнике. – 1973. – Вып. 82. – С. 60–-64.
4. Пересыпкин, С.Е. Внецентренное сжатие бетонных элементов с учётом влияния поперечных сил (расчёт по деформированной схеме на основе блочной модели) : дис. ... канд. техн. наук / С.Е. Пересыпкин. – СПб., 1995. – 124 л.
5. Починок, Ю.В. Блочная деформационная модель в расчётах железобетонных стержневых изгибаемых элементов с трещинами : дис. ... канд. техн. наук : 05.23.01 / Ю.В. Починок. – Ростов н/Д, 2004. – 241 л.
6. Бровкина, М.В. Прикладные методы расчёта прочности и деформативности изгибаемых железобетонных элементов блочной структуры : дис. ... канд. техн. наук : 05.23.01 / М.В. Бровкина. – СПб., 2004. – 194 с.
7. Croce, P. Numerical Simulation of the Behavior of Cracked Reinforced Concrete Members [Electronic resource] / P. Croce, P. Formichi // Materials Sciences and Applications, 5, 883-894. – 2014. – Mode of access: http://dx.doi.org/10.4236/msa.2014.512090.
8. Borosnyói, A. Models for Flexural Cracking in Concrete: The State of the Art. / A. Borosnyói, G.L. Balázs // Structural Concrete. – 2005. – № 6. – P. 53–62.
9. Casanova, A. Bond Slip Model for the Simulation of Reinforced Concrete Structures / A. Casanova, L. Jason, L. Davenne // Engineering Structures. – 2012. – № 39. – P. 66–78. – Mode of access: http://dx.doi.org/10.1016/j.engstruct.2012.02.007.
10. Тур, В.В. Расчетная модель сопротивления изгибу многопустотных плит перекрытий безопалубочного формования в составе платформенных стыков зданий / В.В. Тур, А.Д. Лазовский // Вестник Полоцкого государственного университета. Серия F, Строительство. Прикладные науки. – 2016. – № 16. – С. 108–119.
11. CEB-FIB Model Code for Concrete Structures. – 2010.