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SUFFICIENT CONDITIONS FOR THE SOLVABILITY OF THE PROBLEM OF SYNTHESIS OF NON-RESONANT SYSTEMS WITH LOCALLY INTEGRABLE COEFFICIENTS

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In this paper it is proved that the problem of synthesis of non-resonant linear non-stationary control system with locally integrable and integrally bounded coefficients and disturbance is solved if the upper general exponent of Bohl of the system (7) is globally controllable, i.e if this linear system has the property of uniform global quasi-attainability or the property of uniform full controllability.

Consider the non-stationary control system with disturbance

$$\dot{x} = A(t)x + B(t)u + w(t), \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad w \in \mathbb{R}^n, \quad t \geqslant 0, \tag{1}$$

and locally integrable and integrally bounded [1, p. 252] matrix coefficients *A* and *B*. In this system $u = u(t)$ is measurable and bounded control, $w = w(t)$ is measurable and bounded vector-function with external disturbance.

For all $t \geqslant 0$ denote

$$f(t) = B(t)u(t) + w(t).$$

Then the system (1) can be written as

$$\dot{x} = A(t)x + f(t), \quad x \in \mathbb{R}^n, \quad f \in \mathbb{R}^n, \quad t \geqslant 0. \tag{2}$$

Definition 1 [2] *Equation* (2) is *called a resonance*, if there is an initial condition $x(t_0) = x_0$ and bounded at $t \geqslant t_0$ the function $f(t)$, that the solution of $x(t)$ generated non-boundary on the half-line $t \geqslant t_0$; *equation is not a resonant or non-resonant* called sustained in the sense of bounded input - bounded output.

It is known [2, p.295], that a necessary and sufficient condition for the stability of the equation (2) in the sense of bounded input - bounded output is the exponential stability of the system

$$\dot{x} = A(t)x, \quad x \in \mathbb{R}^n, \quad t \geqslant 0, \tag{3}$$

i.e. on valid

Theorem 1 [2, p. 295] *Equation* (1) *non-resonance then and only then, when the system* (3) *is exponentially stability.*

Definition 1 it follows that the resonant case there are initial conditions that the solutions generated by them are unbounded with bounded control actions. Therefore, the resonance system difficult to implement, resulting in the construction of control systems is usually primarily the problem of ensure non-resonance.

For the system (1) we take as a control u vector function

$$u(t) = U(t)x + v(t), \tag{4}$$

where U and v – some measurable $(m \times n)$ - matrix and $(m \times 1)$ - vector respectively.

Than system (1) can be written as

$$\dot{x} = (A(t)x + B(t)U(t))x + B(t)v + w(t), \quad x \in \mathbb{R}^n, \quad v \in \mathbb{R}^m, \quad w \in \mathbb{R}^n, \quad t \geqslant 0, \tag{5}$$

Definition 2 [2, p. 293] The problem of finding a measurable and bounded $(m \times n)$ -matrix of function $U(t)$, $t \geqslant 0$, at which the closed-loop system (5) with a control (4) is non-resonant (has only bounded solutions with measurable and bounded input actions in $v(t)$ and external disturbances $w(t)$) is *the problem of synthesis of non-resonant system.*

Theorem 1 implies that this problem is solvable if there exists a measurable bounded control

$$U = U(t), \quad t \geqslant 0, \tag{6}$$

what system

$$\dot{x} = (A(t)x + B(t)U(t))x, \quad x \in \mathbb{R}^n, \quad t \geqslant 0, \tag{7}$$

is exponentially stability.

It is well known (see e.g. [3, p.61]), that for the exponential stability of linear non-stationary system (3) (and, therefore, (5)) corresponds to the upper special (general exponent)

$$\Omega^0(A) = \lim_{T\to+\infty} \frac{1}{T} \sup_{k\in\mathbb{N}} \ln \| X((k+1)T, kT) \|$$

(Here $X(t,s), \ t,s \geqslant 0,$ is Cauchy matrix of system (3), and $\|\cdot\|$ are in brackets to demonstrate how its operator matrix (spectral) norm), pioneered by P. Bohl [4], and, much later, but independently of it, K.P. Persidsky [5]. This exponent provides for Cauchy matrix of system (3) uniform bound

$$\| X(t,s) \| \leqslant D_\varepsilon \exp((\Omega^0(A) + \varepsilon)(t-s)),$$

where $D_\varepsilon > 0$ constant depending on the ε, and therefore negativity means that there is a uniform asymptotic (exponent) stability [3, p. 61].

Upper general exponent Bohl $\Omega^0(A)$ (also, e.g., as well as the characteristic exponents of Lyapunov $\lambda_i = \lambda_i(A) \in \mathbb{R},\ i = \overline{1,n},$ irregularities coefficients of Lyapunov, Grobman, Perron, correctness properties and reducibility of systems [3,p.46-80] and others) they represent many of the asymptotic invariants of linear differential systems (3), i.e. variables or properties that are saved under Lyapunov's transformations.

Definition 3 [3, p. 57] *Transformation of Lyapunov of the system* (3) is called a linear transformation $y = L(t)x,$ square $(n \times n)$ - matrix is a matrix $L(t)$ whose for every $t \geqslant 0$ is reversible, piecewise continuously differentiable and thus satisfies the condition:

$$\sup_{t\geq 0} \| L(t) \| + \sup_{t\geq 0} \| L^{-1}(t) \| + \sup_{t\geq 0} \| \dot{L}(t) \| < +\infty.$$

Definition 4 [3, p. 182-183] *The problem of global control of asymptotic invariant* $i(A+BU)$ *of the system* (7) is to find such a measurable and bounded control (6), that the system (7) with the this control will have a pre-assigned value of this invariant. For example, considering this problem as an asymptotic invariant $i(A+BU)$ the upper general exponent of Bohl $\Omega^0(A+BU)$ system (7), we obtain a problem of global control of the upper general exponent of Bohl.

In the case of a positive decision of the latter problem, we have an opportunity with the help of some measurable and bounded control action (6) provides a negative exponent $\Omega^0(A+BU)$ of the system (7) (moreover, can choose this exponent value itself), and, thus, the exponential stability of the system (7), which means that non-resonantly of the equation (5). Therefore, there is a sufficient condition for the solvability of the problem of synthesis of non-resonant system:

Theorem 2 *If the upper general exponent of Bohl of the system (7) is globally controllable, then the problem of synthesis of non-resonant system (5) is solvable.*

Consider linear control system (1) without the presence of disturbance $(w(t) \equiv 0, \ t \geqslant 0)$

$$\dot{x} = A(t)x + B(t)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad t \geqslant 0. \tag{8}$$

Definition 5 [6,7] *The system* (8) is *uniformly full controllable* if there are numbers $\sigma > 0$ and $\gamma > 0,$ that for any $t_0 \geqslant 0$ and $x_0 \in \mathbb{R}^n$ there is a measurable and bounded control $u:[t_0, t_0+\sigma] \to \mathbb{R}^m,$ for all $t \in [t_0, t_0+\sigma]$, satisfying the inequality $\| u(t) \| \leqslant \gamma \| x_0 \|$ and carrying the vector of the initial position $x(t_0) = x_0$ (8) to zero on this segment.

Initially V. A. Zaytsev systems (8) with piecewise continuous and bounded matrix coefficients *A* and *B* and piecewise continuous [8] and the bounded control (6) has been established following

Theorem 3 [8] *If the system* (8) *with piecewise continuous and bounded coefficients is uniformly full controllable, then the upper general exponent of Bohl of the system* (7) *is globally controllable.*

On the basis of this theorem, and **Theorem 2** we obtain

Corollary 1 *If the system* (8) *with piecewise continuous and bounded coefficients is uniformly full controllable, then the problem of synthesis of non-resonant system* (5) *is solvable*.

For the same systems (8) with a locally integrable and integrally bounded coefficients, which are discussed in this article, A. A. Kozlov and I.V. Ints in [9] has been proved for the following two-dimensional case

Theorem 4 [9] *Let* $n = 2$. *If the system* (8) *is uniformly full controllable, then the upper general exponent of Bohl of the system* (7) *is globally controllable*.

From it follows from **Theorem 2**, as above, directly follows

Corollary 2 *Let* $n = 2$. *If the system* (8) *with locally integrable and integrally bounded is uniformly full controllable, then the problem of synthesis of non-resonant system* (5) *is solvable.*

In the case of an arbitrary phase space dimension $n > 2$, for the systems (8) with a locally integrable and integrally bounded coefficients global controllability of the upper general exponent of Bohl succeeded in establishing [9] with a stronger than uniform full controllability, condition - the evenly global quasi-attainability of the system (7).

Definition 6 [9]. *The system* (7) *has the property of uniform global quasi-attainability* if for some $T > 0$ whenever $t_0 \geqslant 0$ and any $r > 0$ and $0 < \rho \leqslant 1$ there orthogonal $(n \times n)$ - matrix $F = F([t_0, t_0 + T]; r, \rho)$ and is independent of the t_0 value of $\theta = \theta(r, \rho) > 0$, such that for any upper triangle $(n \times n)$ -Matrix H with positive diagonal elements, satisfying the inequalities $\| H - E \| \leqslant r$ and $\det H \geqslant \rho$, there is a measurable and bounded on the interval $[t_0, t_0 + T]$ control $U = U(t)$, satisfying for all $t \in [t_0, t_0 + T]$ assessment $\| U(t) \| \leqslant \theta$ and guarantees for the Cachy $X_U(t, s)$ matrix system (7) the equality $X_U(t_0 + T, t_0) = X(t_0 + T, t_0) FHF^{-1}$.

There is a

Theorem 5 [9]. *If the system* (8) *is uniformly globally quasi-attainable, then the upper general exponent of Bohl of the system* (7) *is globally controllable.*

Then, by **Theorem 2** and the last statement is executed

Corollary 3. *If the system* (8) *is uniformly globally quasi-attainable, the problem of synthesis of non-resonant system* (5) *is solvable*.

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REFERENCES

1. Былов, Б.Ф. Теория показателей Ляпунова и её приложения к вопросам устойчивости / Б.Ф. Былов, Р.Э. Виноград, Д.М. Гробман. — М., 1966. —576 с.
2. Гайшун, И.В. Введение в теорию линейных нестационарных систем / И.В. Гайшун. — Минск, 1999. — 409 с.
3. Макаров, Е.К. Управляемость асимптотических инвариантов нестационарных линейных систем / Е.К. Макаров, С.Н. Попова. — Минск : Беларус. навука, 2012. — 408 с.
4. Bohl, P. Über Differentialgleichungen / P. Bohl // J. reine und angew. Math. — 1913. — Bd.144. — S. 284–318.
5. Персидский, К.П. Об устойчивости движения по первому приближению / К.П. Персидский // Матем. сборник. — 1933. — Т. 40, № 3. — С. 284–292.
6. Тонков, Е.Л. Критерий равномерной управляемости и стабилизация линейной рекуррентной системы / Е.Л. Тонков // Дифференц. уравнения. – 1979. – Т.15, № 10. – С. 1804–1813.
7. Kalman, R.E. Contribution to the theory of optimal control / R.E. Kalman // Boletin de la Sociedad Matematika Mexicana. — 1960. — Vol.5, № 1. — P. 102–119.
8. Зайцев, В.А. Об управлении показателями Ляпунова и о λ-приводимости / В.А. Зайцев // Вестник Удмуртского ун-та. — 2000. — № 1. — С. 35–44.
9. Kozlov, A. A. On the global Lyapunov reducibility of two-dimensional linear systems with locally integrable coefficients / A.A Kozlov, I.V. Ints // Differential Equation. — V. 52. — 2016. — №. 6. — P. 699–721.