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ON THE STABILIZABILITY OF TWO-DIMENSIONAL LINEAR SYSTEMS WITH LOCALLY INTEGRABLE COEFFICIENTS AND OBSERVER

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The system of the asymptotic measure of condition for the two-dimensional linear non-stationary control system and observer has been constructed in the assumption of Lebesgue locally integrability and integrally boundness of its coefficients on a positive semi-axis, sufficient conditions of the uniform stabilizability of the asymptotic identificator and with it of the two-dimensional linear system with locally integrable and integrally bounded coefficients and observer has been obtained in this report.

Let $\mathbb{R}^n$ – a vector Euclidean space of dimension n with norm $\| x \| = \sqrt{x^T x}$ for each $x \in \mathbb{R}^n$ (the symbol ^{T} means the operation of transposition); M_{mn} – a space of real $(m \times n)$-matrix with the spectral operator norm

$$\| A \| = \max_{|x| \leqslant 1} \frac{\| Ax \|}{\| x \|},$$

i.e. with norm induced by the Euclidean norm in the spaces $\mathbb{R}^m$ and $\mathbb{R}^n$; $M_n := M_{nn}$.

Let us consider a linear non-stationary control system

$$\dot{x} = A(t)x + B(t)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad t \geqslant 0, \tag{1}$$

with observer

$$y = C^T(t)x, \quad y \in \mathbb{R}^n. \tag{2}$$

We will assume that matrix functions $A(\cdot)$, $B(\cdot)$ и $C(\cdot)$ belong to the class of Lebesgue locally integrable and integrally bounded coefficients [1, c. 252] matrix function, i.e. such that unequalities

$$\sup_{t \geq 0} \int_t^{t+1} \| A(\tau) \| \, d\tau < a < \infty, \quad \sup_{t \geq 0} \int_t^{t+1} \| B(\tau) \| \, d\tau < b < \infty,$$

$$\sup_{t \geq 0} \int_t^{t+1} \| C(\tau) \| \, d\tau < c < \infty$$

are true. Also we will assume that system (1) has the property of uniform full controllability.

Definition 1 [2]**.** *The system* (1) *is called uniformly fully controllable if numbers* $\sigma > 0$ *and* $\gamma > 0$ *exist such that for each* $t_0 \geq 0$ *and* $x_0 \in \mathbb{R}^n$ *measure and bounded control* $u : [t_0, t_0 + \sigma] \to \mathbb{R}^m$ *exist for every* $t \in [t_0, t_0 + \sigma]$ *such that satisfies the unequality* $\| u(t) \| \leq \gamma \| x_0 \|$ *and transfer the vector of the initial condition* $x(t_0) = x_0$ *of the system* (1) *to the null on this time segment.*

Also let us consider the system (1), (2) with null-control, i.e system

$$\dot{x} = A(t)x, \quad y = C^T(t)x, \quad x \in \mathbb{R}^n, \quad y \in \mathbb{R}^k, \quad t \geqslant 0, \tag{3}$$

that has the property of uniform full observability.

Definition 2 [3, p.304]. *The system* (3) *is called uniformly fully observable if number* $\sigma > 0$ *exist such that for each* $t_0 \geq 0$ *every vector of the initial condition* $x(t_0) = x_0 \in \mathbb{R}^n$ *can be defined by observer by the only way on time segment* $t \in [t_0, t_0 + \sigma]$.

Let us construct a new system using the system (1), (2) and y (output of this system)

$$\dot{\hat{x}} = A(t)\hat{x} + V(t)(y(t) - C^T(t)\hat{x}) + B(t)u, \quad \hat{x} \in \mathbb{R}^n, \tag{4}$$

where $\hat{x}(t)$ - measure of condition of the system (1), (2). *System (4) is called the system of asymptotic measure of condition (asymptotic identificator)*. Let us take control u as a linear feedback

$$u = U(t)\hat{x}. \tag{5}$$

Rewriting the system (1), (2), (4) with the control (5) we get $(2n)$-dimensional system

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A(t) & B(t)U(t) \\ V(t)C^T(t) & A(t)+B(t)U(t)-V(t)C^T(t) \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}. \tag{6}$$

Let $\tilde{x} := x - \hat{x}.$ With the non-degenerate transformation of variables

$$\begin{pmatrix} x \\ \dot{\tilde{x}} \end{pmatrix} = \begin{pmatrix} E & 0 \\ E & E \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

let's reduce the system (6) to the form

$$\begin{pmatrix} \dot{x} \\ \dot{\tilde{x}} \end{pmatrix} = \begin{pmatrix} A(t)+B(t)U(t) & -B(t)U(t) \\ 0 & A(t)-V(t)C^T(t) \end{pmatrix} \begin{pmatrix} x \\ \tilde{x} \end{pmatrix}. \tag{7}$$

Definition 3 [4, p. 61]. *Upper general Bohl exponent* Ω^0 *of the homogenious system*

$$\dot{x} = A(t)x, \quad t \geqslant 0 \tag{8}$$

is a number

$$\Omega^0(A) = \lim_{T \to +\infty} \frac{1}{T} \sup_{k \in \mathbb{N}} \ln \| X((k+1)T,\ kT) \|,$$

where $X(t,\ s),\ t,\ s \geqslant 0$ *is Cauchy matrix of the system* (8).

Definition 4 [2]. *The system* (7) *is called uniformly stabilized if for each* $\alpha > 0$ *measure and bounded controls* $U(\cdot)$ *and* $V(\cdot)$ *can be constructed such that relation* $\Omega^0_{U,V} > -\alpha.$ *is true.*

Definition 5 [4, p. 259]. *The system* (1) *has the property of global Lyapunov reducicility if measure and bounded control* (5) *can be constructed as a linear feedback such that the corresponding closed-loop system*

$$\dot{x} = (A(t)+B(t)U(t))x, \quad x \in \mathbb{R}^n, \quad t \geqslant 0 \tag{9}$$

is asymptotically equivalent (kinematic similar) [4, p. 56-57] *to arbitrary preassigned system*

$$\dot{z} = K(t)z, \quad z \in \mathbb{R}^n, \quad t \geqslant 0, \tag{10}$$

with locally integrable and integrally bounded matrix of coefficients $K(t),\ t \geqslant 0.$

Thus means [4, p. 57-58] that linear transformation

$$z = L(t)x, \quad t \geqslant 0,$$

which is called Lyapunov transformation exists and reduce system (9) into system (10). Here $(n \times n)$-matrix $L(t)$ (Lyapunov matrix) for each $t \geqslant 0$ is bouded, reversible, has integrally bounded and wherein satisfies the following

$$\sup_{t \geq 0} \|L(t)\| + \sup_{t \geq 0} \|L^{-1}(t)\| + \sup_{t \geq 0} \int_t^{t+1} \|\dot{L}(\tau)\|\, d\tau < +\infty.$$

Using the method had been proposed by V. Zaitsev [5], the problem of constructing control as a linear feedback for system (1), (2) of a small dimensions of phase space that assure uniform stabilization for system (7) is considered in this report. Thereby, main result of this report are

Theorem 1. *Let* $n = 2,\ m \in \{1, 2\}$. *If system* (1) *with locally integrable and integrally bounded coefficients has the property of uniform full controlability, system* (3) *with the same coefficients has the property of uniform full observability, then system* (7) *is uniformly stabilized.*

In the report [6] the following theorem 2 was proved by A. Kozlov and I. Ints

Theorem 2. *Let* $n = 2,\ m \in \{1, 2\}$. *If system* (1) *with locally integrable and integrally bounded coefficients has the property of uniform full controlability, system* (3) *with the same coefficients has the property of uniform full observability, then the corresponding closed-loop by feedback* (5) *system* (9) *has the property of global Lyapunov reducibility.*

Basing on thus, theorem 3 have been proved

Theorem 3. *Let* $n = 2,\ m \in \{1, 2\}$. *If system* (1) *with locally integrable and integrally bounded coefficients has the property of uniform full controlability, system* (3) *with the same coefficients has the property of uniform full observability, then for each locally integrable and integrally bounded matrix functions* $P:[0, +\infty) \to M_n$ *and* $Q:[0, +\infty) \to M_n$ *measure and bounded controls* $U(\cdot)$ *and* $V(\cdot)$ *exist such that system* (7) *with theese controls is reduced to the following system*

$$\begin{pmatrix} \dot{z} \\ \dot{\tilde{z}} \end{pmatrix} = \begin{pmatrix} P(t) & G(t) \\ 0 & Q(t) \end{pmatrix} \begin{pmatrix} z \\ \tilde{z} \end{pmatrix}, \quad z \in \mathbb{R}^n, \quad \tilde{z} \in \mathbb{R}^n, \quad t \geqslant 0$$

with some locally integrable and integrally bounded matrix $G(t)$.

Particularly if $P(t) \equiv P$ and $Q(t) \equiv Q$ for each $t \geqslant 0$, P and Q are arbitrary preassigned real $(n \times n)$-matrix, the following is true

Corollary 1. *Let* $n = 2,\ m \in \{1, 2\}$. *If system* (1) *with locally integrable and integrally bounded coefficients has the property of uniform full controlability, system* (3) *with the same coefficients has the property of uniform full observability, then for each preassigned real* $(n \times n)$*-matrix*

$$P \in M_n \quad \text{and} \quad Q \in M_n$$

measure and bounded controls $U(\cdot)$ *and* $V(\cdot)$ *can be found such that the system* (7) *of asymptotic measure of condition with theese controls can be reduced to the system*

$$\begin{pmatrix} \dot{z} \\ \dot{\tilde{z}} \end{pmatrix} = \begin{pmatrix} P & G \\ 0 & Q \end{pmatrix} \begin{pmatrix} z \\ \tilde{z} \end{pmatrix} \tag{11}$$

with the some constant matrix $G \in M_n$.

Thence for the two-dimensional linear control system with locally integrable and integrally bounded coefficients (1) the foolowing statement implies about stabilization:

Corollary 2. *Let* $n = 2,\ m \in \{1, 2\}$. *If system* (1) *with locally integrable and integrally bounded coefficients has the property of uniform full controlability, system* (3) *with the same coefficients has the property of uniform full observability, then for each* $\kappa > 0$ *measure and bounded controls* $U(\cdot)$ *and* $V(\cdot)$ *and Lyapunov transformation*

$$\begin{pmatrix} x \\ \dot{x} \end{pmatrix} = L(t) \begin{pmatrix} z \\ \tilde{z} \end{pmatrix},$$

exist and reduce the system (7) *of asymptotic measure of condition with theese controls to the system* (11) *with constant matrix of coefficients* D *such that unequality* $\mathrm{Re}\lambda_i(D) \leqslant -\kappa < 0$ *is true for each eigenvalues* $\lambda_i(D),\ i = 1, \ldots, 2n,$ *of matrix* D.

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