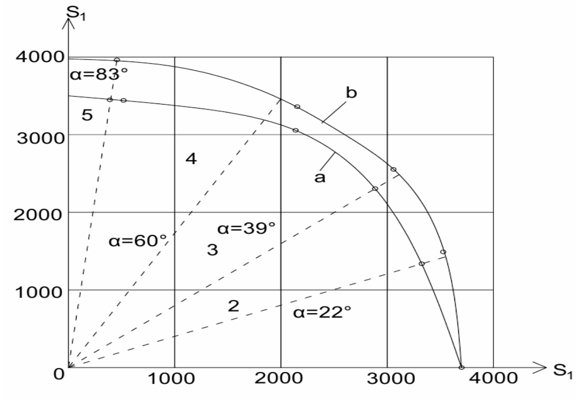

Fig. 7. Yield curve and isomodular line for specimens subjected to plastic prestraining in tension, at $Э^{p}_{i}$=0.05

In order to estimate errors that are caused by neglecting the anisotropy considered in the paper, experimental data and theoretical predictions obtained by the theory of kinematical hardening and by the theory accounting for the Bauschinger effect were compared, by considering and disregarding a change of modulus K. The comparison was made for two combined loading paths which caused the greatest discrepancy between theoretical results and experimental data.

Plastic strain components were determined in graphic form using the curve σ — ε^{p} obtained experimentally from data of tensile specimens. The results are presented in Figs. 1-7. The effect of the variable K on the accuracy of analytical predictions is most pronounced in Fig. 5, in which plastic strains found experimentally and evaluated theoretically by accounting for the Bauschinger effect are compared for a specimen subjected to initial shearing strain $э^{P}$ = 2.7 per cent.

The strain components obtained in accordance with the mentioned theories even allowing for the described plastic strain induced anisotropy, deviate considerably from those found experimentally (this applies specifically to the theory of kinematical hardening). No better agreement can, however, be expected as we take into account, only one of the factors influencing inaccuracy of the available flow theories. And in view of some reasons for this inaccuracy of the flow theories can be given, i.e. lack of coincidence of the yield loci obtained experimentally and theoretically. Also the influence of the stress deviator on the plastic deformation process was not considered in any of the mentioned theories. The accuracy of these theories is also influenced by the fact that the deviators of stress and plastic strain are not coaxial even under proportional loading as found by several investigators.

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STRAIN CALCULATION FOR COMPRESSED CURVED BAR UNDER COMBINED APPLICATION OF THERMAL EXPOSURE AND LOAD

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Differential equation of the axis of a compressed curved bar with unspecified ends fixity under combined application of thermal exposure and load has been obtained. Application of the obtained equation for strain calculation for a beam, hinged at its ends, has been demonstrated.

It is usual practice to carry out analysis of structures in relation to load exposure and temperature exposure separately and subsequently summate the determined stress-strain state (SSS) parameters in accordance with the principle of superposition. But this approach is true for linearly strained structures only. In case of flexible structures geometrical nonlinearity and structure strain calculation must be taken into account. Such issues as flexible compressed curved bars strain calculation are investigated in depth. Herein, referenced documents [1], [2], [3] may be noted. However, differential equations, set out in these documents and underlying compressed curved bars strain calculation, do not account for thermal exposure effect on SSS parameters variation due to geometrical nonlinearity consideration.

Strain calculation of a linear elastic bar of symmetrical uniform cross-section with unspecified ends fixity, limiting all the motions of the end sections in full or partially, is considered. The bar is exposed to the action of the axial force *N*, unspecified transverse load and thermal exposure (fig.1).

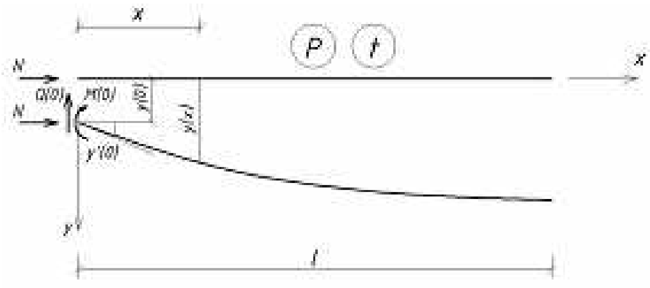

Fig. 1

Transverse load and thermal exposure are shown in fig.1 with conventional letters *P* and *t*.

The thermal exposure is characterized by two independent values – internal temperature increment $\Delta t_в$ and surface temperature increment $\Delta t_н$, as well as by the value, dependent on them – temperature increment on the bar axis Δt_o. Higher temperature is taken as the internal temperature. The rate of change of the temperature increment in the transverse section height *h* is determined from the following formula:

$$\Delta t' = \frac{\Delta t_в - \Delta t_н}{h}$$

The stress-strain state (SSS) of the bar in any section is characterized by deflection $y(x)$, slope $y'(x)$, bending moment $M(x)$, transverse force $Q(x)$ and constant axial force *N*. SSS of the bar at the origin of the coordinates is characterized by the initial parameters y_0, y'_0, M_0, Q_0 and depends on the conditions of the section fixture in this position.

As was shown above [4] curvature of the bar distortion, resulting from the thermal exposure, is described by the equation

$$\left(\frac{1}{\rho}\right)_T = -\alpha\Delta t'. \quad (1)$$

Curvature of the bending, produced by the load action, is described by the equation, known from the strength of materials theory:

$$\left(\frac{1}{\rho}\right)_P = \pm\frac{M}{EI}, \quad (2)$$

where E – modulus of elasticity of the bar construction material; I – moment of inertia of the bar cross-section about the axis, perpendicular to the bending plane; M – bending moment, occurring in any section.

Thereafter the curvature of the bar under joint action of temperature and load shall be obtained by summation of (1) and (2) and with consideration of the law of signs in the coordinate system, shown in fig. 1 takes the following form

$$\frac{1}{\rho} = -\frac{M}{EI} - \alpha\Delta t'. \quad (3)$$

The moment value with consideration of the axial force is described by the expression

$$M(x) = M(0) + Q(0)x + N\left[y - y(0)\right] + M_P, \quad (4)$$

where M_P – bending moment, occurring in any section from the transverse load action.

When we plug approximation for the line curvature in the left-side part of the equation (3), we obtain differential equation of the compressed curved bar under joint action of load and thermal exposure

$$y'' + k^2 y = -\frac{M(0) + Q(0)x - Ny(0) + M_P}{EI} - \alpha\Delta t', \tag{5}$$

where $k^2 = \frac{N}{EI}$.

The equation obtained (5) is an ordinary non-homogeneous differential equation of second order with constant coefficients and its solution is given by

$$y = y_1 + y_2.$$

Where

$$y_1 = A\sin(kx) + B\cos(kx)$$

is a general solution of the homogeneous differential equation, obtained from (5) and

$$y_2 = -\frac{M(0) + Q(0)x - Ny(0) + M_P + EI\alpha\Delta t'}{k^2 EI}$$

is a specific solution of the equation (5).

Expressing arbitrary constants through initial parameters we obtain the following solution of the equation (5)

$$y = y(0) + \frac{y'(0)}{k}\sin kx - \frac{M(0) + M_P + EI\alpha\Delta t'}{k^2 EI}(1 - \cos kx) - \frac{Q(0)}{k^3 EI}(kx - \sin kx). \tag{6}$$

The solution obtained describes the deflections, occurring in the bar. On having differentiated (6) once with respect to *x*, we'll obtain an equation for slopes

$$y' = y'(0)\cos kx - \frac{M(0) + M_P + EI\alpha\Delta t'}{kEI}\sin kx - \frac{Q(0)}{k^2 EI}(1 - \cos kx). \tag{7}$$

Let us demonstrate application of the obtained formulas (6), (7) for compressed curved bar strain calculation with particular scheme of the bar supporting and determination of SSS parameters. Let us consider a simple hinged beam under combined action of temperature, axial force *N* and transverse force *P*, applied in the middle of the beam span. The scheme of the beam in the state of strain is presented in Figure 2.

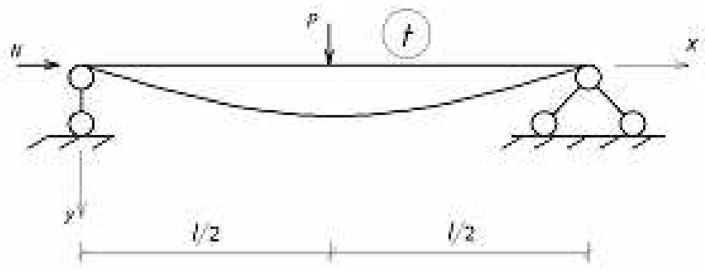

Fig. 2

In accordance with the conditions of the beam fixture at the origin of the coordinates the initial parameters are characterized by the following values

$$y_0 = 0,\ y_0' \neq 0,\ M_0 = 0,\ Q_0 = \frac{P}{2}. \tag{8}$$

In view of (8) the equation for deflections (6) shall take the following form

$$y = \frac{y'(0)}{k}\sin kx - \frac{\alpha\Delta t'}{k^2}(1 - \cos kx) - \frac{P}{2k^3 EI}(kx - \sin kx). \tag{9}$$

It follows from the conditions of the beam right end fixture (x=*l*) that the deflection at this end is equal to zero $y(l) = 0$. Consequently

$$y'(0) = \frac{P}{2EI}\frac{kl - \sin kl}{k^2 \sin kl} + \alpha\Delta t'\frac{(1 - \cos kl)}{k\sin kl}. \tag{10}$$

In view of (10-18) deflections of a simple hinged beam will be described by the expression

$$y=\frac{Pl^3}{2EI}\frac{1}{\nu^3}\left[\frac{\nu-\sin\nu}{\sin\nu}\sin\nu\xi-(\nu\xi-\sin\nu\xi)\right]+ +\alpha\Delta t' l^2\frac{1}{\nu^2}\left[\frac{1-\cos\nu}{\sin\nu}\sin\nu\xi-(1-\cos\nu\xi)\right] \quad (11)$$

and the expression for bending moments takes the following form

$$M=\frac{Pl}{2}\left\{\xi+\frac{1}{\nu}\left[\frac{\nu-\sin\nu}{\sin\nu}\sin\nu\xi-(\nu\xi-\sin\nu\xi)\right]+ +\frac{2EI\alpha\Delta t'}{Pl}\left[\frac{1-\cos\nu}{\sin\nu}\sin\nu\xi-(1-\cos\nu\xi)\right]\right\}. \quad (12)$$

Expressions (11) and (12) are set down with the use of non-dimensional parameter of the axial force $\nu = kl$ and non-dimensional abscissa of the section $\xi=\frac{x}{l}$.

Setting in (11) $x=0.5l$, let's find the maximum deflection under combined action of temperature, axial force N and transverse force P

$$y_{\max}=y_{\max}^{P}\left[24F_1(\nu)+\frac{48}{\pi^2}\alpha\beta\frac{\Delta t}{\rho}F_2(\nu)\right], \quad (13)$$

where $y_{\max}^{P}=\frac{Pl^3}{48EI}$ – maximum deflection under transversal force P; $\beta=\frac{l}{h}$ – parameter, taking account of beam span to beam section height ratio; $\Delta t=\Delta t_{в}-\Delta t_{н}$ – thermal exposure parameter; $\rho=\frac{P}{N_{кр}}$ – parameter of the level of the transverse loading on the beam in fractions of the beam crippling load $N_{кр}=\frac{\pi^2 EI}{l^2}$. Functions $F_1(\nu)$ and $F_2(\nu)$, included in (13), take account of the axial force effect and take the form of

$$F_1(\nu)=\frac{1}{\nu^3}\left(\frac{\nu-\sin\nu}{2\cos\frac{\nu}{2}}-\left(\frac{\nu}{2}-\sin\frac{\nu}{2}\right)\right),$$

$$F_2(\nu)=\frac{1}{\nu^2}\left(\frac{1-\cos\nu}{2\cos\frac{\nu}{2}}-\left(1-\cos\frac{\nu}{2}\right)\right).$$

Setting in (12) $x=0.5l$, let's find the maximum moment under combined action of temperature, axial force N and transverse force P

$$M_{\max}=M_{\max}^{P}\left\{1+\nu^2\left[12F_1(\nu)+\frac{4}{\pi^2}\alpha\beta\frac{\Delta t}{\rho}F_2(\nu)\right]\right\},$$

where $M_{\max}^{P}=\frac{Pl}{4}$ – maximum deflection under transversal force P.

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