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ABOUT THE GENERALIZATION OF THE DEFINITION OF UNIFORM GLOBAL CONTROLLABILITY FOR THE SYSTEMS WITH LOCALLY INTEGRABLE COEFFICIENTS

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The classical definition of uniform global controllability was formulated for linear controllable systems with coefficients from the space L_2. *It was defined in terms of the Cauchy's matrix of such systems with zero control and matrix at control, and therefore was an effective tool for solving many applied problems. The above concept of locally integrable systems is generalized in this paper.*

Let R^n – n-dimensional Euclidean vector space with norm $\| x \|= \sqrt{x^T x}$ (here the symbol T means the transpose operation of a vector or matrix). Consider a linear non-stationary control system

$$\dot{x} = A(t)x + B(t)u, \quad x \in R^n, \quad u \in R^m, \quad t \geq 0. \tag{1}$$

with locally integrable and integrally bounded coefficients A and $B.$ If the matrix B belongs also to the space integrable with a square matrix function, then we have a

Definition 1 [1]. *The system* (1) *is called uniformly globally controllable (by Kalman) if there are the number* $\sigma > 0$ *and* $\alpha > 0,$ *what for all* $t \geq 0$ *and* $\xi \in R^n \setminus \{0\}$ *is fulfilled the inequality*

$$\xi^T \int_{t_0}^{t_0+\sigma} X(t_0,\tau) B(\tau) B^T(\tau) X^T(t_0,\tau)\, d\tau\, \xi \geq \alpha \| \xi \|^2 . \tag{2}$$

where $X(t,s),\ t,s \geq 0,$ *– Cauchy matrix of system* (1) *with zero control.*

Obviously, this definition is expressed in terms of the Cauchy matrix $X(t,s)$ of system (1) with zero control and the matrix B at control. Since the Cauchy matrix is absolutely continuous, then the existence of the integral in (2) boils down to the question of integrability of the square matrix $B.$ If the latter is not integrable with square, this integral does not exist, and, therefore, for systems (1) with such matrix B denition 1 does not make sense. E. L. Tonkov [2] introduced a different definition of uniform global controllability, equivalent definition 1 in the case of the integrability of the square matrix B which also holds for the case when the matrix at control does not belong to the space of integrable with the square of functions.

Definition 2 [2]. *The system* (1) *is called uniformly globally controllable (by Tonkov) if there are the number* $\sigma > 0$ *and* $\gamma > 0,$ *what for all* $t \geq 0$ *and* $x_0 \in R^n$ *there is a measurable and bounded control* $u:[t_0, t_0+\sigma] \to R^n$ *for all* $t \in [t_0, t_0+\sigma]$ *satisfies the inequality* $\| u(t) \| \leq \gamma \| x_0 \|$ *and translating the vector of the initial condition* $x(t_0) = x_0$ *of system* (1) *to zero on this segment.*

This definition fully describes the content, believe that the concept of «uniform global controllability». However, it does not provide effective means for the establishment of uniform global controllability of system (1), such, what has the definition 1. The question therefore arises of finding the conditions similar to (2) on the basis of which we could talk about the presence of the linear system (1) with locally integrable and integrally bounded coefficients properties of uniform global controllability. The study of this question was devoted to the article [3], which for system (1) introduced the concept of properties of $H(\theta),$ definition is equivalent to 2.

This article provides a generalization of the definition of uniform global controllability in terms of the Cauchy's matrix and of the matrix at control for linear control system (1) with locally integrable and integrally bounded coefficients, which is equivalent to definition 2 (by Tonkov) and definition 1(by Kalman) (for the case when the coefficients A and B also belong to the space L_2).

For any number $t \geq 0$ and an arbitrary matrix functions $D(t) = \{d_{ij}(t)\}_{i=\overline{1,m},\, j=\overline{1,n}}$ denote by $\operatorname{sign} D(t)$ matrix function of the form $\operatorname{sign} D(t) = \{\operatorname{sign} d_{ij}(t)\}_{i=\overline{1,m},\, j=\overline{1,n}},$ here $\operatorname{sign}:[0,+\infty) \to \{-1,0,1\}$ is the Signum function, i.e. function of the form

$$\operatorname{sign} f(s) = \begin{cases} 1 & \text{for all } s \text{ such, that } f(s) > 0, \\ 0 & \text{for all } s \text{ such, that } f(s) = 0, \\ -1 & \text{for all } s \text{ such, that } f(s) < 0. \end{cases}$$

Definition 3. *The system* (1) *is called uniformly globally controllable if there are the number* $\sigma > 0$ *and* $\delta > 0$, *what for all* $t \geq 0$ *and* $\xi \in R^n \setminus \{0\}$ *is fulfilled the inequality*

$$\xi^T \int_{t_0}^{t_0+\sigma} X(t_0,\tau)B(\tau) \cdot \operatorname{sign}\left(B^T(\tau)X^T(t_0,\tau)\right) d\tau\, \xi \geq \delta \| \xi \|.$$

where, as before, $X(t,s)$, $t,s \geq 0$, *– Cauchy matrix of system* (1) *with zero control.*

Fair of the following theorems:

Theorem 1. *Let the matrix* B *is integrable with a square matrix function. The system* (1) *with locally integrable and integrally bounded coefficients uniformly globally controllable by definition* 1 *(by Kalman) then, and only then when it is uniformly globally controllable in the sense of definition* 3.

Theorem 2. *Let the matrix* B *is not integrable with square. The system* (1) *with locally integrable and integrally bounded coefficients uniformly globally controllable by definition* 2 *(by Tonkov) then, and only then when it is uniformly globally controllable in the sense of definition* 3.

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FPGA USE FOR DIGITAL SIGNAL PROCESSING

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This article considers the possibility of increasing the performance of digital signal processing (DSP) using parallel computing techniques. The features of using FPGAs are given. The advantages of using FPGA to DSP processors are compared.

Introduction. The term "digital signal processing» (DSP) covers a wide area, the boundaries of which are difficult to determine. Often the hallmark of digital signal processors is the presence of hardware support for the "multiply-accumulate" operation (MAC). MAC is reduced to the computation of the sum of products:

$$y = \sum_{i=0}^{n} k_i \cdot x_i. \tag{1}$$

This amount can be considered approximately equal to the integral

$$\int_t k(t) \cdot x(t) dt, \tag{2}$$

to which a large set of mathematical methods of signal analysis is reduced [1].

One of the main requirements for digital information processing systems is high performance. To achieve high speed computation is possible with the help of the techniques of parallel calculations, which in most cases use integrated circuits (ICs), such as Field Programmable Gate Array (FPGA).

ICs of this type are programmable logic array (PLA), between the elements of which electrically switched connections are laid. This allows you to configure the individual components and establish communication between them by loading the FPGA data stream including the required circuit and switching nodes. As a result the required digital circuitry is generated out of the available resources in the composition of the PLA. This digital circuitry can be easily modified. Modern FPGAs have a large amount of resources, reaching millions of equivalent logic gates that make up hundreds of thousands of logic cells, which allows the design of digital devices of any complexity [2].

Since its introduction FPGA devices have been positioned as superior signal processors in price / performance ratio. But compared to the relatively cheap microcontrollers and signal processors, FPGAs do not jus-