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SUPERRESOLUTION OF RADIATION SOURCES ON THE BASIS OF KEYPON'S METHOD

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The results of theoretical analysis of super resolution of light sources using antenna arrays with the help of Keypon's method have been presented. Digital techniques to ensure effective noise reduction features and high-quality processing of radar signals have been applied. The function of the angular resolution of authorization objects is analyzed. The results of studies for one or two light sources are given. Recommendations are given to reduce the computational complexity of algorithms. The results of the research can be used in radio position finding and radiolocation.

One of the most important tasks of the radar is to measure the angular coordinates of the radiation source of the desired signal, which is based on the determination of the direction of arrival of electromagnetic waves emitted or reflected by the target.

The relevance of the study is determined by the fact that for modern radar stations severe restrictions on the weight, dimensions and power consumption are imposed, which excludes the use of high-power transmission devices and limits the size of the web array.

Keypon's method was proposed in 1969 to resolve the spectral components of the discrete spectrum [1]. Using the analogy of the frequency and spatial spectra method was proposed for estimating the angles of arrival of signals using an antenna array.

Model signals received from the N antenna array elements may be represented as follows:

$$Z = \sum_{k=1}^{J} a_k S(\varphi_k) + X , \quad (1)$$

where J – the number of discrete sources;

a_κ, φ_κ – complex amplitude and angle of arrival of the waves corresponding to index k;

$S(\varphi_\kappa)$ – N-dimensional vector signals received from the power grid with the number k;

X – N-dimensional noise vector own antenna array elements;

Z – N-dimensional vector of the received signal.

The model assumes that the complex amplitudes of waves of different sources are statistically independent, i.e.

$$< a_k a_m^* > = \begin{cases} \sigma_k^2 (k = m); \\ 0 (k \neq m). \end{cases} \quad (2)$$

Since the discrete sources, the signals in the elements of the array for each source are assumed to be correlated (correlation coefficient equal to unity). This means that the signal vector $S(\varphi_\kappa)$ of each source is deterministic and describes both the nature of the wave front and the configuration of the antenna array. Noises in its elements are assumed to be uncorrelated and equal in power. The problem is formulated as follows: it is necessary to find the weight vector W, which minimizes the average power output of the antenna array with the proviso that, for some angle of arrival φ transmission ratio is fixed and the lattice is, for example, a unity. Mathematically, this task can be written as follows:

$$\min_{w} < \left| W^H Z \right|^2 > \text{ provided } W^H S(\varphi) = 1 . \quad (3)$$

It is the task of a conditional extremum.

To solve it, we need to create a functional Lagrange

$$\Phi(W) = < \left| W^H Z \right|^2 > - \chi \left(W^H S(\varphi) - 1 \right). \quad (4)$$

The χ – undetermined Lagrange multiplier.

The first term in (4) can be written as

$$< \left| W^H Z \right|^2 > = W^H < ZZ^H > W = W^H M W, \quad (5)$$

where $M = < ZZ^H >$ – correlation matrix of the signals at the input of the antenna array.

For the adopted model signals (1) it is easy to calculate the correlation matrix under the condition (2).

As a result, we obtain

$$M = \sum_{k=1}^{J} \sigma_k^2 S(\varphi_k) S^H(\varphi_k) + \sigma^2 E. \quad (6)$$

The σ^2 – the average noise power in a single element antenna array.

In view of (5), the expression (4) is transformed into

$$\Phi(W) = W^H MW - \chi\left(W^H S(\varphi) - 1\right). \tag{7}$$

The gradient of the functional equate to zero and obtain the following equation:

$$MW = \lambda S(\varphi). \tag{8}$$

Hence we find the weight vector:

$$W = \chi M^{-1} S(\varphi), \tag{9}$$

where M^{-1} – inverse correlation matrix of the input signals.

Now the weight vector (9) should be substituted to a desired condition (3), and the undetermined factor χ then can be found in the form of

$$\chi = \left[S^H(\varphi) M^{-1} S(\varphi)\right]^{-1}. \tag{10}$$

The final solution of the problem is obtained by (10) substituted into (1). As a result, the weight vector that minimizes the average power output according to Keypon's criterion will have the form

$$W = \frac{1}{S^H(\varphi) M^{-1} S(\varphi)} M^{-1} S(\varphi). \tag{11}$$

In an optimal state, when the weighting coefficients of the antenna array are established in accordance with (11), the average output power is found by substituting (11) into (5). This value is the function of the resolution, which we denote $\eta_1(\varphi)$.

Thus, for Keypona's method the resolving function is found to be

$$\eta_1(\varphi) = \frac{1}{S^H(\varphi) M^{-1} S(\varphi)}. \tag{12}$$

An average power output can be measured. Therefore, this value is of interest in terms of the angular resolution of sources.

1. Suppose that the space has a single source. This example is useful to consider, although in case of a single source the question of angular resolution does not make sense. In this case the correlation matrix (6) takes the form

$$M = \sigma_1^2 S(\varphi_1) S^h(\varphi_1) + \sigma^2 E. \tag{13}$$

Figure 1 shows the function (12). The calculations were performed for the linear equidistant antenna array with the number of elements of $N = 16$ and a half-wave length between elements.

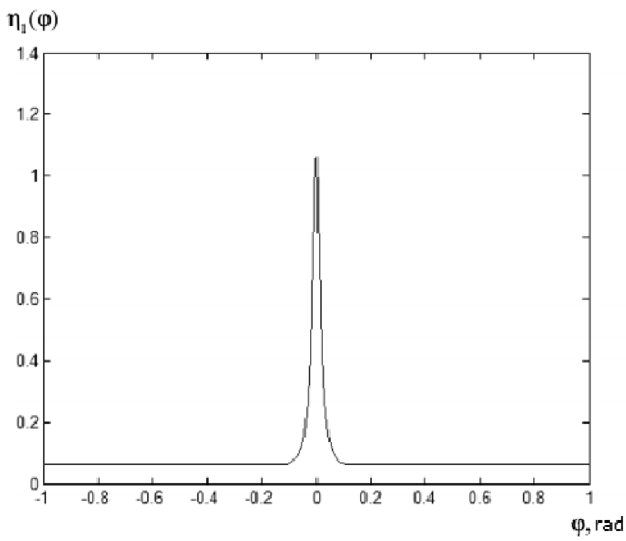

Fig. 1. The average power output of the antenna array of the angle φ

It was assumed that a plane wave coming from the source in a direction normal to the antenna array, i.e. $\varphi_1 = 0$. The average power of the desired signal and the noise floor in each element taken respectively $\sigma_1^2 = 1$, $\sigma^2 = 1$.

2. Let us now assume that the space has two sources. Then, the correlation matrix of the received signals (6) has the form

$$M = \sigma_1^2 S(\varphi_1) S^h(\varphi_1) + \sigma_2^2 S(\varphi_2) S^h(\varphi_2) + \sigma^2 E. \quad (14)$$

Assume that both sources have the same average power in each antenna array element. We also assume that the ratio of signal power to the power source noise floor in each cell is equal to unity, i.e. $\sigma_1^2 = 1$, $\sigma^2 = 1$; the angles of arrival of waves are assumed to be equal: $\varphi_1 = \pi/64$, $\varphi_2 = -\pi/64$.

As before, assume that the measurement of angles of arrival of the waves is performed by linear equidistant array of 16 elements ($N = 16$) and a half-wave length interelemental d/λ =0,5. Let's apply Keypon's method (12), which does not require a priori knowledge of the number of sources, and is based only on the knowledge of the correlation matrix of the input signals. The results are shown in Figure 2.

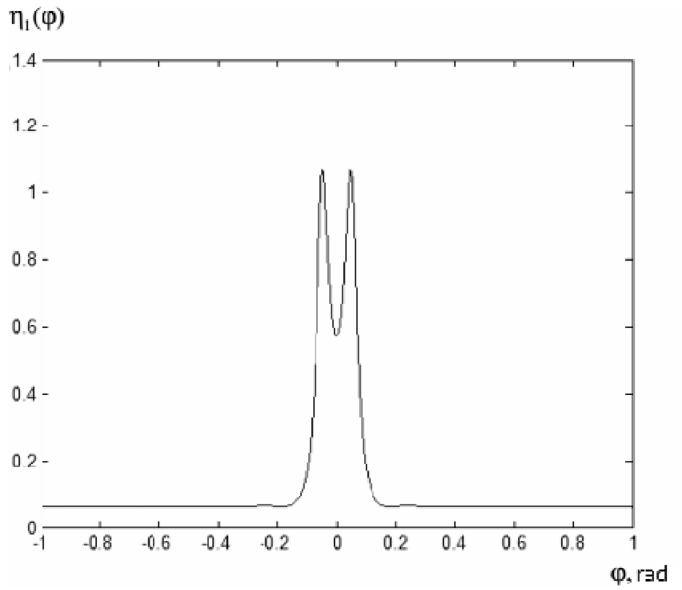

Fig. 2. The average power output of the antenna array depending on the angle φ with $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 1$

Keypon's method has the following advantages:

– does not require a priori knowledge of the number of radiation sources;
– less computational complexity of the algorithm than the maximum likelihood method;
– has a higher resolution than the maximum likelihood method;
– less influence of random amplitude and phase errors.

However, the method has some disadvantage:

– the worst accuracy rates than the method of "thermal noise";
– this method does not allow to estimate the number of radiation sources.

The analysis of Keypon's method for super resolution has been performed. There have been developed recommendations based on a symmetric matrix that help reduce the computational complexity of algorithms, which allows to simplify the digital implementation of these algorithms. The research results can be used in radar systems.

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