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ON THE PROPERTY OF PARTIAL UNIFORM GLOBAL ATTAINABILITY OF LINEAR CONTROL SYSTEMS

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In this paper we consider the problem of global Lyapunov reducibility of linear differential systems and shows the main results available to solve this problem. With entered our concept of partial uniform global attainability, we obtain a solution of this problem for three-dimensional systems with discontinuous and rapidly varying coefficients.

Consider a linear non-stationary control system

$$\dot{x} = A(t)x + B(t)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad t \geqslant 0, \tag{1}$$

with locally integrable and integrally bounded matrix coefficients A and B. Closing the system (1) with the control defined in the form of a linear feedback

$$u = U(t)x, \tag{2}$$

where U is a measurable and bounded $(m \times n)$ – matrix, we obtain a closed system

$$\dot{x} = (A(t) + B(t)U(t))x, \quad x \in \mathbb{R}^n, \quad t \geqslant 0, \tag{3}$$

coefficients are also locally integrable and integrally bounded. Along with (3) we also consider an arbitrary system

$$\dot{z} = C(t)z, \quad z \in \mathbb{R}^n, \quad t \geqslant 0, \tag{4}$$

with measurable integrally bounded matrix coefficients C.

The problem of global Lyapunov reducibility of linear system (3) is to construct for a system (1) a measurable and bounded control (2) that the linear system (3), closed this control will be asymptotically equivalent [1, c. 56 - 57] to system (4). This means [1, c. 57 - 58] that there will be a linear transformation relating system (3) and (4)

$$x = L(t)z,$$

matrix which satisfies

$$\sup_{t \geqslant 0}(\| L(t) \| + \| \dot{L}(t) \| + \| L^{-1}(t) \|) < \infty.$$

There $\| \cdot \|$ – is spectral (operator) norm of matrices [2, c. 355], i.e. matrix norm induced by the Euclidean norm.

The problem of global Lyapunov reducibility was formulated [3] by representatives of the Izhevsk school of mathematics Tonkov E.L. and Zaitsev V.A. in the early 90th years of the 20th century. Professor Tonkov E.L. proposed to seek a solution to this problem assuming uniform complete controllability of system (1).

Definition 1 [3, 4]. The system (1) is *uniformly completely controllable* if there are numbers $\sigma > 0$, $\gamma > 0$, that for any $t_0 \geqslant 0$ and $x_0 \in \mathbb{R}^n$ in interval $[t_0, t_0 + \sigma]$ there is a measurable and bounded control u at all $t \in [t_0, t_0 + \sigma]$ satisfying the inequality $\| u(t) \| \leqslant \gamma \| x_0 \|$ and transforming the vector of the initial condition $x(t_0) = x_0$ of the system (1) to zero on this interval.

In 1999, using this approach by Professors Makarov E.K. and Popova S.N. has been proved [5] global Lyapunov reducibility of a two-dimensional system (3) with piecewise continuous and bounded coefficients in the case of piecewise uniform continuity of the matrix B.

Definition 2 [6]. Let M_{nm} is space of real $n\times m$ matrices with spectral (operator) norm; then the matrix function $B:[0,+\infty)\to M_{nm}$ *piecewise uniformly continuous* on the positive semiaxis, if it satisfies the following conditions:

1) the function B is piecewise continuous and bounded on the positive semiaxis;

2) there exists a number $\Delta>0$, that the length of each interval of continuity I_j $(j\in J\subset\mathbb{N})$ of the function B satisfies the inequality $|I_j|\geq\Delta$;

3) for any $\varepsilon>0$ there exists such $\delta=\delta(\varepsilon)>0$, that for each $j\in J$ and for all $t,s\in I_j$, satisfying the inequality $|t-s|\leqslant\delta$, the relation $\|B(t)-B(s)\|\leqslant\varepsilon$. is true.

Remark. In view of Definition 2, the system (1) with a piecewise uniformly continuous matrix of the control can be called slowly varying systems.

Later, on the basis of the results of [5] Popova S.N. shown [6] that for n-dimensional ω-periodic system (1) (which obviously is a slowly varying system) uniform complete controllability is sufficient for the global Lyapunov reducibility of the corresponding a closed system (3).

It should also be noted that in solving the problem of global Lyapunov reducibility [1, 3, 5-7] by classics condition piecewise uniform continuity of the matrix B plays an important role. The refusal of this condition leads to unlimited growth of the desired control $U(t)$ for $t\to\infty$, but this is unacceptable on the basis of formulation of the problem. In this connection there is the question of transferring the results obtained by the classics on the system (3) with discontinuous and rapidly oscillating coefficients, i.e. solution to the problem of the global Lyapunov reducibility for systems (3) with locally integrable and integrally bounded coefficients.

We also note that problem of global Lyapunov reducibility of linear system (3) is a generalization of the problem of global control Lyapunov's exponents [3] of this system, which is responsible for its asymptotic stability, i.e. availability of the system (3) property of the global Lyapunov reducibility is a sufficient condition for global controllability of Lyapunov characteristic exponents of the system. In the turn generalization of the global Lyapunov reducibility of (3) is the problem of uniform global reachability [7].

Definition 3. [7]. Let $M_n:=M_{nn}$. For any $r\geqslant 1$ we denote the set of matrices $G(r):=\{H\in M_n:\det H\geqslant 1/r,\|H\|<r\}$.

The system (3) is called *uniformly globally attainable,* if for some $T>0$ for any $r\geqslant 1$ there exists a number $d=d(r)>0$, that for each matrix $H\in G(r)$ and random $t_0\geqslant 0$ in the interval $[t_0,t_0+T]$ there exists a piecewise continuous and bounded control U, satisfying $\|U\|\leqslant d$ for all $t\in[t_0,t_0+T]$ in which for the Cauchy matrix $X_U(t,s)$, $t,s\geqslant 0$ of the system (3) with this control is provided equality $X_U(t_0+T,t_0)=H$.

Individual results for solving the problem of uniform global attainability were obtained by V.A. Zaitsev [7], E.K. Makarov and S.N. Popova [5]. However, the general solution of this problem to date has been found.

It is known [7] that the presence of the system (3) properties of uniform global attainability is a sufficient condition for its global Lyapunov reducibility. In this study, we found that for three-dimensional systems a sufficient condition for the global Lyapunov reducibility is a weaker condition than the uniform global attainability – availability of the system (3) properties of partial uniform global attainability.

Definition 4. The system (3) is *partially uniformly globally attainable set of matrices $G(r)$ with respect to subsets $\mathfrak{F}\subseteq GL_n(\mathbb{R})$ of non-singular square $(n\times n)$- matrices*, if for some $T>0$ at all $r\geqslant 1$ and $t_0\geqslant 0$ there exists a matrix $F\in\mathfrak{F}$, that for any matrix $H\in G(r)$ there is a number $d=d(r)>0$ and a control U, satisfying the inequality $\|U\|\leqslant d$ for all $t\in[t_0,t_0+T]$, which for the Cauchy matrix $X_U(t,s)$, $t,s\geqslant 0$ of the system (3) with this control satisfies $X_U(t_0+T,t_0)=FHF^{-1}$.

Definition 5. Let $G_\Delta(r)$ is the set of upper triangular matrices of $G(r)$. Then the system (3) is *partially uniformly globally attainable* if it is partially uniformly globally attainable set $G_\Delta(r)$ with respect to a plurality of orthogonal $(n\times n)$- matrices $O(n)$.

Thus we proved

Theorem 1. *Let* $n=3$, $m\in\{1,2,3\}$. *If the system (3) with locally integrable and integrally bounded coefficients partially uniformly globally attainable, then it is globally Lyapunov reducible.*

Also, we found that the property of uniform complete controllability of system (1), in turn, is a sufficient condition for the partial uniform global attainability of the corresponding closed system (3), i.e. proved

Theorem 2. *Let* $n = 3$, $m \in \{1,2,3\}$. *If the system (1) with locally integrable and integrally bounded coefficients uniformly completely controllable, then the system (3) has the property of partial uniform global attainability.*

From theorems 1 and 2, the main result of this work

Theorem 3. *Let* $n = 3$, $m \in \{1,2,3\}$. *If the system (1) with locally integrable and integrally bounded coefficients are uniformly completely controllable, the corresponding closed system (3) is globally Lyapunov reducible.*

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PULSE MODULATION FOR ULTRA-WIDEBAND COMMUNICATION SYSTEMS

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The basic types of pulse modulation used for ultra-wideband communication systems are considered. The interactions between a modulated signal and additive white Gaussian noise are analyzed. The most noiseproof and optimum types of modulation for practical implementation are defined.

The fundamental direction is to increase the channel capacity for wireless telecommunication systems. Data transmission rate is proportional to the width of signal spectrum. So, for example, sufficient channel bandwidth makes 8 kHz for a voice signal, 180 kHz for high-quality transmission of music, 5 MHz for video [1]. The ideal case is to have a universal communication channel which can be used to transmit any kind of information, which implies a large bandwidth. Channel transmission ultra-wideband (UWB) communication systems or "impulse radio" has a similar property.

Information transfer in UWB communication systems is carried out by means of short pulses of sub-nanosecond duration. One of the major advantages of UWB systems is the lack of direct interference signal propagating with his reflections on various objects. Short pulses extend through different obstacles, because attenuation happens not in the entire range. UWB systems can operate with low average total transmit power due to the high effective gain, so these systems do not interfere with other wireless devices operating in the same band. Since the energy of the UWB signal is distributed in a wide range, the task of detection and interception becomes almost impossible. [2]

For the "mpulse radio" can be used various types of modulation: PPM, PAM, OOK, BPSK [3-4]. Therefore, the organization of UWB communication system appears task of choosing the optimal type of pulse modulation. The solution must satisfy the following conditions:

- high noise immunity;
- ease of implementation of the modulating demodulation equipment;
- high data rate.

In the case of Pulse Position Modulation (PPM) a logical "1" and a logical "0" encoded different positions relative to the reference pulse (Fig. 1 (*a*)). On-Off Keying (OOK) implies the presence of short pulses at "1" and their absence at "0" (Fig. 1 (*b*)). In case of Pulse Amplitude Modulation (PAM) "1" and "0" coded varying amplitude of ultra-short pulses (Fig.1 (*c*)). In the case of Binary Phase Shift Keying (BPSK) a logical "0" and a logical "1" corresponds to a certain phase of the pulse: 0^0 or 180^0 (Fig. 1 (*d*)) [4].