

Let us assume that we need to multiply 11111 and 12323. We divide the second factor into primitives. These are numbers 1, 2, 3, 2, 3. This means that we need to calculate the following intermediate results:

- 11111 x 1 = 11111;
- 11111 x 2 = 22222;
- 11111 x 3 = 33333.

Then we rewrite the primitives in reverse order and perform addition of the intermediate results with a shift:

Primitives: 3, 2, 3, 2, 1.

Summation: (33333) + (22222 << 1) + (33333 << 2) + (22222 << 3) + (11111 << 4) = 33333 + 222220 + 3333300 + 22222000 + 111110000 = 136920853.

Further optimization of the algorithm is possible. The essence of optimization is to change the principle of the partition of the second factor into primitives. Primitives will be repeated sequences of numbers. Let us take the previous example and perform multiplication (11111 and 12323). We divide the second factor into primitives. This is the sequence: 1, 23, and 23. Now, we need to calculate the following intermediate results:

- 11111 x 1 = 11111;
- 11111 x 23 = 255553.

Summation: (255553) + (25553 << 2) + (11111 << 4) = 136920853.

**Exponentiation.** For exponentiation a number which is raised should be multiplied by itself as many times as the exponent. However, the exponent  $n > 3$  already requires large computing power. To optimize exponentiation so-called "binary exponentiation is used". The essence of the method is similar to the suggested multiplication algorithm (intermediate calculations are used). The essence of the method is the following:

Provide the exponent in the form of a sum of powers of 2.

Calculate the number of intermediate squares.

Multiply them.

Let us take a detailed look at the method. Let's assume that we need to calculate  $x^n$ .

We provide the exponent in the form of a sum of powers of 2. Each number can be written as:

$$n = \sum_{i=0}^n a_i + 2^i = a_0 * 2^0 + a_1 * 2^1 + a_2 * 2^2 + \dots + a_{n-1} * 2^{n-1} + a_n * 2^n$$

where  $a \in \{0;1\}$ .

Next, we compute the intermediate squares, i.e. we find  $x^2, x^4$ , etc.

Finally, we multiply the intermediate squares together. It is worth noting that the multiplication is carried out only with those intermediate results, where  $a_i$  in the expansion of the exponent is 1.

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#### MULTIPATH PROPAGATION PROBLEM ANALYSIS IN DATA TRANSMISSION SYSTEMS

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*This article deals with the problem of multipath propagation and electromagnetic wave when it is incident on a layered medium.*

*For the selection of tangible objects on the background of the environment in practice is usually used the reflective characteristics serving tool to optimize the electrical parameters of the probing signal.*

The fall of a plane electromagnetic wave on a flat boundary of two semi-infinite anisotropic media 1 and 2 is illustrated in Figure 1 for the horizontal (perpendicular) polarization when the electric field vector is perpendicular to the plane  $E_0$  fall, and vertical (parallel) polarization when the electric field vector lies in the plane  $E_0$  fall.

Fresnel reflection coefficient for horizontally polarized waves (in the coordinate system  $x, y, z$ , shown in Figure 1).

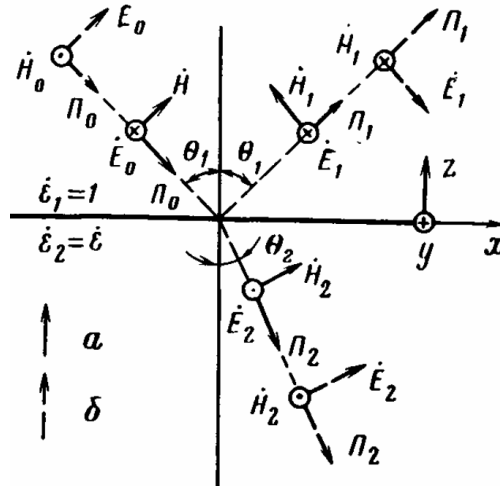


Fig. 1. The fall of a plane wave at the interface with the horizontal (a) and vertical (b) polarizations

$$\dot{R}_{(1-2)r} = \frac{\dot{E}_1}{\dot{E}_0} = \frac{\dot{W}_2 \cos \theta_1 - \dot{W}_1 \cos \theta_2}{\dot{W}_2 \cos \theta_1 + \dot{W}_1 \cos \theta_2} \quad (1)$$

Transmission coefficient

$$\dot{T}_{(1-2)r} = \frac{\dot{E}_2}{\dot{E}_0} = \frac{2\dot{W} \cos \theta_1}{\dot{W}_2 \cos \theta_1 + \dot{W}_1 \cos \theta_2} \quad (2)$$

and the complex characteristic impedance (characteristic impedance) of the environment

$$\dot{W} = \frac{\dot{E}}{H} = \sqrt{\frac{\mu_0 \dot{\mu}}{\epsilon_0 \dot{\epsilon}}} = 120\pi \sqrt{\frac{\dot{\mu}}{\dot{\epsilon}}} \quad (3)$$

Where in a nonmagnetic medium

$$\dot{W} = \frac{120\pi}{\sqrt{\dot{\epsilon}}} = \frac{120\pi}{\dot{n}} \quad (4)$$

As for the angles of incidence and refraction, it should be borne in mind that, according to the Snellius law for absorbing media

$$\sin \theta_2 = \frac{\dot{\gamma}_1}{\dot{\gamma}_2} \sin \theta_1 = \sqrt{\frac{\dot{\epsilon}_1 \dot{\mu}_1}{\dot{\epsilon}_2 \dot{\mu}_2}} \sin \theta_1 \quad (5)$$

is complex.

In case when  $\epsilon_1, \mu_1$  are real, the true value of the angle of refraction is equal to

$$\theta_2 = \text{arccctg}[\text{Re}(\text{ctg} \theta_2)]. \quad (6)$$

For vertically polarized wave in the coordinate system  $x, y, z$  in Figure 1 we have the following expressions

$$\dot{R}_{(1-2)B} = \frac{\dot{E}_{1x}}{\dot{E}_{0x}} = \frac{\dot{W}_2 \cos \theta_2 - \dot{W}_1 \cos \theta_1}{\dot{W}_2 \cos \theta_2 + \dot{W}_1 \cos \theta_1} \quad (7)$$

$$\dot{T}_{(1-2)B} = \frac{\dot{E}_{2x}}{\dot{E}_{0x}} = \frac{2\dot{W}_2 \cos \theta_2}{\dot{W}_2 \cos \theta_2 + \dot{W}_1 \cos \theta_1} \quad (8)$$

It should be noted that sometimes the reflection coefficient for vertical polarization is determined by the ratio of the corresponding magnetic vectors. The sign changes  $\dot{R}_{(1-2)B} = \dot{H}_1 / \dot{H}_0 = -\dot{E}_1 / \dot{E}_{0x}$ .

For vertical drop ( $\theta_1 = \theta_2 = 0$ ) and non-magnetic media expressions (1), (2), (7) and (8) take the following forms:

$$\dot{R}_{1-2} = \frac{\sqrt{\dot{\epsilon}_1} - \sqrt{\dot{\epsilon}_2}}{\sqrt{\dot{\epsilon}_1} + \sqrt{\dot{\epsilon}_2}}, \tag{9}$$

$$\dot{T}_{1-2} = \frac{2\sqrt{\dot{\epsilon}_1}}{\sqrt{\dot{\epsilon}_1} + \sqrt{\dot{\epsilon}_2}}. \tag{10}$$

Let us consider the multilayered model, which takes into account the layered permafrost, alternating layers of sand and clay, and so on. Let us represent it in the form  $n - 1$  layers with uniform impedance  $\dot{Z}_2, \dot{Z}_3 \dots \dot{Z}_n$  (complex permittivities are respectively  $\dot{\epsilon}_2, \dot{\epsilon}_3 \dots \dot{\epsilon}_n$ ) located between two semi-infinite media 1 and  $n + 1$ .

Multilayered model is presented in Figure 2.

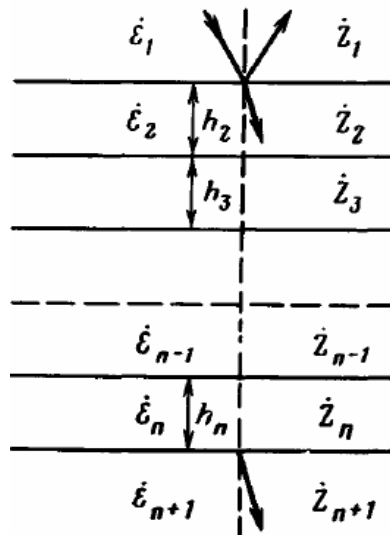


Fig. 2. Multilayered model

Let us assume that there is only a layer with index  $n$ , and the medium with index  $n - 1$ , as well as that with  $n + 1$ , extends indefinitely. Then, the input impedance of the upper boundary layer of index  $n$  is

$$\dot{Z}_{gx}^n = \frac{(\dot{Z}_{n+1} + \dot{Z}_n) + (\dot{Z}_{n+1} - \dot{Z}_n)e^{-2\dot{\gamma}_n h_n}}{(\dot{Z}_{n+1} + \dot{Z}_n) - (\dot{Z}_{n+1} - \dot{Z}_n)e^{-2\dot{\gamma}_n h_n}} \dot{Z}_n. \tag{11}$$

Since while moving through the boundary layer tangential components of the electric and magnetic fields do not change, impedances are saved as well. Therefore, the impedance at the lower boundary of the layer with index  $n - 1$  coincides with the input impedance  $\dot{Z}_{gx}^n$  of the layer with index  $n$ .

Gradual transition from one layer to another enables us to find the input impedance  $\dot{Z}_{gx}^3$  at the upper boundary layer with index 3. The addition of another layer (with index 2) allows us to calculate the input impedance at the upper boundary for the entire system of layers in the form of:

$$\dot{Z}_{gx}^{(n-1)} = \frac{(\dot{Z}_{gx}^n + \dot{Z}_{n-1}) + (\dot{Z}_{gx}^n - \dot{Z}_{n-1})e^{-2\dot{\gamma}_{n-1} h_{n-1}}}{(\dot{Z}_{gx}^n + \dot{Z}_{n-1}) - (\dot{Z}_{gx}^n - \dot{Z}_{n-1})e^{-2\dot{\gamma}_{n-1} h_{n-1}}} \dot{Z}_{n-1}. \tag{12}$$

Therefore, the final reflection coefficient of the layers is

$$\dot{R} = \frac{(\dot{Z}_{6x}^2 - \dot{Z}_1)}{(\dot{Z}_{6x}^2 + \dot{Z}_1)} \quad (13)$$

Let's consider a two-layer medium, which  $n = 3$ . The thicknesses of the 1st and the 2nd layers are, respectively,  $h_2$  and  $h_3$ , and their propagation constants are  $\dot{\gamma}_2, \dot{\gamma}_3$ . This medium coefficient (13) is calculated using the formula (12), if we substitute the value  $\dot{Z}_{6x}^3$  calculated by formula (11):

$$\dot{Z}_{6x}^3 = \frac{(\dot{Z}_4 + \dot{Z}_3) + (\dot{Z}_4 - \dot{Z}_3)e^{-2\dot{\gamma}_3 h_3}}{(\dot{Z}_4 + \dot{Z}_3) - (\dot{Z}_4 - \dot{Z}_3)e^{-2\dot{\gamma}_3 h_3}} \dot{Z}_3 \quad (14)$$

In a particular case when the thicknesses of all layers are equal to zero ( $h_2 = h_3 = 0$ ), the input impedance of the system  $\dot{Z}_{6x}^2$  is equal to the impedance  $\dot{Z}_4$ , as it should be in case of absence of layers.

The above formulas enables expression (14) to take the following form

$$\dot{R} = \frac{\dot{R}_{1-2} + w e^{-2\dot{\gamma}_2 h_2}}{1 + \dot{R}_{1-2} + w e^{-2\dot{\gamma}_2 h_2}} \quad (15)$$

where

$$w = \frac{\dot{Z}_{6x}^3 - \dot{Z}_2}{\dot{Z}_{6x}^3 + \dot{Z}_2} = \frac{\dot{R}_{2-3} + \dot{R}_{3-4} e^{-2\dot{\gamma}_3 h_3}}{1 - \dot{R}_{2-3} \dot{R}_{3-4} e^{-2\dot{\gamma}_3 h_3}} \quad (16)$$

This method makes it possible to analyze the layered media, specifically, to determine the physical properties of the layer (e.g., dielectric constant) as well as their number. This method can find its application in geology and geolocation to detect all kinds of deposits.

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#### MODEL MANAGEMENT SYSTEM THAT IS BASED ON DATA PROCESSING AND ANALYSIS IN REAL TIME WITH INSTRUMENTS MATLAB

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*The block diagram and the practical implementation of model management system based on the processing and analysis of data in real time are considered. Interfaces mutual exchange of data streams of MATLAB and external equipment were considered. The example of using the data in real time with application of the system objects MATLAB is given.*

Communication and control systems are the most important aspects of work with hardware and software. Direct communication with equipment and peripheral device allows you to quickly test and verify algorithms, and to carry the necessary changes in projects under development. MATLAB software provides a lot of different ways to interact with sophisticated equipment and peripherals.