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ABOUT ONE LEGAL ROUTE IN THE SPACE OF THE SQUARE MATRIXES

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The task of the construction of one legal route in the space of the square matrixes of the n-th order is considered in the article. This route connects an identity matrix with a certain matrix the last column of which is a preassigned n-dimensional vector-column codirectional with the last column of the identity matrix.

Let's consider Euclidean n-dimensional space R^n. Let's denote the vectors of the canonical orthonormalized basis of this space by means of e_i, $i=\overline{1,n}$. We take up the space M_n along with the vectorial space R^n. The space M_n is the space of the square real-valued matrixes of an order n with the spectral (operator) norm [1, c.355], i.e. the norm, which is induced on M_n by Euclidean norm in the space R^n. We give the following definition.

Let $\xi_1,\dots,\xi_2$ be the sequence of vectors from the space R^n, where ρ is a certain positive number. The sequence of matrix $P_0,\dots,P_l \in M_n$ is called ρ- *legal route (concerning the sequence of vectors ξ_i), which connects points P_0 and P_l*, if the unequality is in progress $\det P_i \ge \rho$, $i=\overline{0,l}$ and vectors $u_i \in R^n$, $i=\overline{1,l}$ are found, and in every $i=\overline{1,l}$ the following correlations take place

$$P_i - P_{i-1} = \xi_i \cdot u_i^T.$$

A natural number l is called the length of the legal route [2].

The legal route is an auxiliary instrument when we solve tasks of the global controllability of different asymptotical invariants [2–6] of the linear non-stationary control systems of the ordinary differential equations of the following type

$$\dot{x} = A(t)x + B(t)u, \quad x \in R^n, \quad u \in R^m, \quad t \ge 0. \tag{1}$$

So, for example, with the help of the appropriately constructed legal route [2] we got the proofs of the global controllability of Lyapunov`s exponents n-dimensional linear systems (2) with the sectionally evenly continued factors, as well as of two-dimensional [3] and three-dimensional [4, 5] linear systems (1) with local integrable and integrally bounded matrixes of the factors A and B; we set the global controllability of the complete summation of Lyapunov`s invariants [6; 7, c. 281-325] of the two-dimensional linear systems (1) with a continuous and bounded matrix A and with a bounded piecewise uniformly continuous matrix B.

In our research we drew on the method of the construction of the legal route for the square matrixes of the second order [3] and found the legal route in the space M_n of the square matrixes of the n-th order. This route connects an identity matrix with a certain matrix the last column of which is a preassigned n-dimensional vector-column codirectional with the last column of the identity matrix.

The main result of the paper (theorem 2) is based on lemmas 1,2, theorem 1 and its corollary.

Lemma1. *If we have any numbers $0<\delta,\ \beta\le 1$ and arbitrary identity vectors $\xi_i \in R^n$, $i=\overline{1,n}$ with the estimation $|\det[\xi_1,\dots,\xi_n]|\ge\delta$ among vectors $\nu_i \in R^n$, $\|\nu_i\|=1$, $i=\overline{1,n}$ which meet $|\det[\nu_1,\dots,\nu_n]|\ge\beta$, for every $l\in\{1,\dots,n\}$, there can be found at least one vector ν, at which the following correlation is correct*

$$|\det[\xi_1,\dots,\xi_{l-1},\nu,\xi_{l+1},\dots,\xi_n]| \ge \delta\beta/n.$$

Let's denote normalized bases formed from vectors $\nu_i(j)\in R^n$, $i=\overline{1,n}$ for every $j=\overline{1,n}$ through $\mathfrak{B}_j$ i.e. a set of vectors

$$\mathfrak{B}_j = \left\{\nu_1(j), \nu_2(j), ..., \nu_n(j) : \|\nu_i(j)\| = 1,\ \nu_i(j)\in R^n,\ i=\overline{1,n}\right\}.$$

The volume of a system of the vectors (basis) $\mathfrak{B}=\{\nu_1,\nu_2,...,\nu_n\}\subset R^n$ [8, c.260-261] is a non-negative number, which is equal to $|\det\mathfrak{B}| = |\det[\nu_1,\nu_2,...,\nu_n]|$.

Geometrically lemma 1 means that for every two bases $\mathfrak{B}_1$ и $\mathfrak{B}_2$ from the space of R^n, which have a sufficiently large volume, and for every vector $\nu_1 \in \mathfrak{B}_1$ there can always be found at least one vector $\nu_2 \in \mathfrak{B}_2$, so that if we replace ν_1 by ν_2, we get a system of vectors from $\mathfrak{B}_1$, which will be a normalized basis in the R^n This system of vectors has a sufficiently large volume (it depends on the volumes of bases $\mathfrak{B}_1$ и $\mathfrak{B}_2$).

T h e o r e m 1. *We have a canonical basis* e_i, $i=\overline{1,n}$ *of the space* R^n. *For random numbers* $\beta \in (0,1]$ *in every* n *the set of the identity vectors* $\nu_1^{(i)},\nu_2^{(i)},\ldots\nu_n^{(i)} \in R^n$, $i=\overline{1,n}$ *with* $\det[\nu_1^{(i)},\nu_2^{(i)},\ldots\nu_n^{(i)}] \ge \beta$, *ther can be found at least one vector* $w^{(i)}$. *This vector belongs to the* i *-th set and at which the following inequality is correct*

$$|\det[w^{(1)},e_2,e_3,\ldots,e_n]| \ge (\beta/n)^n =: \delta_n, \quad |\det[w^{(1)},w^{(2)},e_3,\ldots,e_n]| \ge \delta_n, \quad \ldots,$$
$$|\det[w^{(1)},w^{(2)},w^{(3)},\ldots,w^{(n)}]| \ge \delta_n.$$

Geometrically theorem 1 means that for the canonical basis of the n-dimensional vectorial space $e_1,e_2,...,e_n \in R^n$ and an ordered sequence of n normalized bases $\mathfrak{B}_i$, $i=\overline{1,n}$ of sufficiently large volumes in every of them there can be found at least one vector $\nu_{j_i}^{(i)} \in \mathfrak{B}_i$, $j_i \in \{1,...,n\}$, . If we coherently replace every vector e_i for $\nu_{j_i}^{(i)} \in \mathfrak{B}_i$, we get a system of the vectors from the canonical basis

$$\mathfrak{B}_i^{'} = \{\nu_{j1}^{(1)},\nu_{j2}^{(2)},...,\nu_{jk}^{(k)},e_{k+1},...,e_n\},\ j_k \in \{1,...,n\}, \quad k=\overline{1,n}$$

This system of the vectors will be normalized bases in the R^n of a sufficiently large volume (however, this system of the vectors will possibly have the direction opposite to that of the canonical basis).

O b s e r v a t i o n. If for vectors $w^{(i)}$, $i=\overline{1,n}$, described in the theorem 1, we put

$$\nu_i := \mathrm{sign}(\det[\nu_1,\nu_2,\ldots,\nu_{i-1},w^{(i)},e_{i+1},\ldots,e_n])w^{(i)}, \quad i=\overline{1,n},$$

we`ll get inequalities

$$\det[\nu_1,e_2,e_3,\ldots,e_n] \ge \delta_n, \quad \det[\nu_1,\nu_2,e_3,\ldots,e_n] \ge \delta_n, \quad \ldots, \quad \det[\nu_1,\nu_2,\nu_3,\ldots,\nu_n] \ge \delta_n.$$

C o r o l l a r y t o t h e o r e m 1. *Matrixes*

$$P_0 := E,\ P_1 := [v_1,e_2,\ldots,e_n],\ P_2 := [v_1,v_2,e_3,\ldots,e_n],\ \ldots,\ P_n := [v_1,v_2,\ldots,v_n],$$

in which vectors ν_i, $i=\overline{1,n}$ *are determined by theorem 1 and observation, conform in the space* $n\times n$ *-matrixes* δ_n *-legal route relative to the sequence of vectors* e_i, $i=\overline{1,n}$, *connecting points* E *и* P_n.

L e m m a 2. *For any numbers* $0<\delta$ � 1 *and* $0<\varphi<\delta/2$ *and arbitrary identity vectors* $\xi_i \in R^n$, $i=\overline{1,n}$ *with the estimation* $|\det[\xi_1,\xi_2,...,\xi_n]| \ge \delta$, *if for the identity vector* $\xi_k' \in R^n$ *the inequality* $\measuredangle(\xi_k',\xi_k) \le \varphi$ *is completed with* $k \in \{1,...,n\}$, *the correlation* $|\det[\xi_1,\xi_2,...,\xi_{k-1},\xi_k',\xi_{k+1},...,\xi_n]| \ge \delta/2$ *is correct.*

Geometrically lemma 2 means that if we have «the departure» of one of the vectors of the normalized basis by a sufficiently little angle measure (i.e. if we replace one of the vectors of the normalized basis for an identity vector situated together with a removable vector in a cone and the cone has a sufficiently little angle measure), the system of vectors which we get will be also the basis. Its volume can change insignificantly.

When we make the legal route and work with vectors of different normalized bases of the space R^n , we`ll reckon that the following operations with these vectors are executable:

1) expansion (shrinkage) of every vectors (i.e. multiplication of the vector by the real number);

2) substitution of every vector from every basis for such two identity vectors of the space R^n which are «deflected» from the replaceable vector by a sufficiently littlle angle measure (i.e. situated together with a replaceable vector in the cone of a sufficient little angle measure) and a certain linear combination of which gives a replaceable vector.

The relevancy of the introduction and feasibility of the given operations on vectors of normalized bases of the space R^n follow from the possibility of their introduction and justification when we solve the main problem – the problem of the global management of Lyapunov`s exponents [3].

T h e o r e m 2. *Suppose that for a random number* $\delta \in (0,1]$ *we get the identity vectors* $\nu_1, \nu_2, ..., \nu_n \in \mathbb{R}^n$ *and* $\det P_0 := \det[\nu_1, \nu_2, \ldots, \nu_n] \geq \delta_0$ *takes place. Then for any non-zero vectors* $h \in R^n$, *which is codirectional with the vector* ν_n, *and every* $i = \overline{1,n}$ there can be *found a vector* w_i^0, *which belongs to the i-th set of the random identity vectors* $w_j(i) \in \mathbb{R}^n$, $j = \overline{1,n}$, *that meets the inequality* $\left|\det[w_1(i), w_2(i), \ldots, w_n(i)]\right| \geq \beta \in (0,1]$, *and the sequences of the numbers* $\alpha_i' \in \mathbb{R}$ *bounded on the module* $|\alpha_i'| \leq (2n/\beta)^n (\|h\|+1) =: \gamma$, $i = \overline{1,n}$, *and* $\sigma_i \in \{1,2\}$, *that if the correlations are correct*

$$\alpha = 2^{3n+2} n (\|h\|+1)(n/\beta)^{2n} \quad \text{and} \quad \varphi \leq \arcsin(1/\alpha),$$

then for every $i = \overline{1,n}$ *and for the vectors* $w_i, w_i' \in R^n$, *the conditions are correct*

$$\|w_i\| = \|w_i'\| = 1, \quad \measuredangle(w_i^0, w_i) \leq \varphi, \quad \measuredangle(w_i^0, w_i') \leq \varphi, \quad i = \overline{1,n},$$

and a certain linear combination forms vector w_i^0, *the following equality takes place*

$$h - \nu_n = \alpha_1' w_1' + \alpha_2' w_2' + \ldots + \alpha_n' w_n'$$

and the following estimations take place

$$\det P_1 := \det[\nu_1 + (-1)^{\sigma_1} \alpha w_1, \nu_2, \ldots, \nu_{n-1}, \nu_n] \geq \delta,$$

$$\det P_2 := \det[\nu_1 + (-1)^{\sigma_1} \alpha w_1, \nu_2, \ldots, \nu_{n-1}, \nu_n + \alpha_1' w_1'] \geq \delta,$$

$$\det P_3 := \det[\nu_1 + (-1)^{\sigma_1} \alpha w_1, \nu_2 + (-1)^{\sigma_2} \alpha w_2, \ldots, \nu_{n-1}, \nu_n + \alpha_1' w_1'] \geq \delta,$$

$$\det P_4 := \det[\nu_1 + (-1)^{\sigma_1} \alpha w_1, \nu_2 + (-1)^{\sigma_2} \alpha w_2, \ldots, \nu_{n-1}, \nu_n + \alpha_1' w_1' + \alpha_2' w_2'] \geq \delta, \quad \ldots,$$

$$\det P_{2n-1} := \det[\nu_1 + (-1)^{\sigma_1} \alpha w_1, \nu_2 + (-1)^{\sigma_2} \alpha w_2, \ldots, \nu_{n-1} + (-1)^{\sigma_{n-1}} \alpha w_{n-1}, \nu_n + \sum_{i=1}^{n} \alpha_i' w_i'] =$$

$$= \det[\nu_1 + (-1)^{\sigma_1} \alpha w_1, \nu_2 + (-1)^{\sigma_2} \alpha w_2, \ldots, \nu_{n-1} + (-1)^{\sigma_{n-1}} \alpha w_{n-1}, h] \geq \delta.$$

REFERENCES

1. Хорн, Р. Матричный анализ / Р. Хорн, Ч. Джонсон. – М.: Мир, 1989. – 655 с.
2. Попова, С.Н. Глобальная приводимость линейных управляемых систем к системам скалярного типа / С.Н. Попова // Дифференц. уравнения. – 2004. – Т. 40, № 1. – С. 41 – 46.
3. Козлов, А.А. Об управлении показателями Ляпунова двумерных линейных систем с локально-интегрируемыми коэффициентами / А.А. Козлов // Дифференц. уравнения. – 2008. – Т. 44, № 10. – С. 1319 – 1335.
4. Козлов, А.А. О существовании линейно-независимых направлений движения для равномерно вполне управляемой трехмерной линейной системы с локально интегрируемыми коэффициентами / А.А. Козлов, А.Д. Бурак // Веснік ВДУ. – 2013. – № 3(75). – С. 29 – 45.
5. Козлов, А.А. Об управлении характеристическими показателями трехмерных линейных дифференциальных систем с разрывными и быстро осциллирующими коэффициентами / А.А. Козлов А.Д. Бурак // Веснік ВДУ. – 2013. – №5 (77) – С. 11 – 31.
6. Макаров, Е.К. О глобальной управляемости полной со- вокупности ляпуновских инвариантов двумерных линейных систем / Е.К. Макаров, С.Н. Попова // Дифференц. уравнения. – 1999. – Т.35, № 1. – С. 97 – 106.
7. Макаров, Е.К. Управляемость асимптотических инвариантов нестационарных линейных систем / Е.К. Макаров, С.Н. Попова. – Минск: Беларус. навука, 2012. – 407 с.
8. Глазман, И.М. Конечномерный линейный анализ / Глазман И. М., Любич Ю.И. – М., 1969. – 476 с.