CORRELATION BINARY IMAGE PROCESSING BASED ON MATRIX FACTORISATION

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Abstract. A novel algorithm for time reduction in binary image processing, namely for computation of correlation between image and object template is proposed. This algorithm is based on direct computation of vector-matrix multiplication with utilisation of binary matrix factorisation approach. Comparison with other algorithms is given and it is shown that our approach allows to reduce time and complexity of this task.

1. INTRODUCTION

The extraction of object location in an image is often based on comparison of image or its fragments with object template [1]. The template is moved through images and is compared with all objects of the image step by step. Quantity of the correlation function is used as estimation of correspondence of the object and template [2].

In correlation approach, one of the most difficult computational procedures is calculation of the correlation function. In paper [3], method for detecting patterns by means of a binary joint transform correlator is presented. The method is based on the compensation of the sign errors introduced in the joint power spectrum by the transfer function of the degradation. Two alternatives to determine the sign of the transfer function are demonstrated: the first is based on an algorithm to extract information from the Fourier spectrum of the blurred image and the second method determines the sign errors by post-processing the correlation.

Correlation functions based on Fourier transformation are usually time consuming for computation [4]. For binary images, computation of correlation function is easier task but it also has limitation for image size, needs the complex data processing and leads to additional computational complexity. It usually takes a big processing time, which is not always acceptable practically.

One of the promising approaches to reduce processing time for matching template with image is to simplify processed images. Such as binary images often contain many `zeros` it can be probably done based on using matrix factorisation technique.

Several algorithms for matrix factorization have been recently proposed [5,6]. Idea of matrix factorisation is based on decomposition of an arbitrary matrix into some simple factor matrices. Choosing optimal size of blocks in which matrix should be decomposed to reduce vector-matrix multiplication computational complexity is given in [7].

This paper analyses how matrix factorisation techniques can be applied for correlation processing of binary images. We propose an algorithm for reduction of complexity and time for correlation binary image processing based on direct computation of vector-matrix multiplication with utilisation of binary matrix factorisation approach.

2. CORRELATION BINARY IMAGE PROCESSING

Correlation approach is based on pixel-to-pixel comparison of analysed image and object template. In this case misalignment will determine the level of correlation function between two images, and the correlation will be maximum at minimum misalignment.

Thus correlation image processing task proposes the correlation function calculation between two images, one of which is utilised as an object template and another as an analysed image. In general, correlation function between images S_{n1} and S_{n2} will be computed as [1]:

$$R_{s,x}[\tau_1,\tau_2]=\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}S_{n1,n2}X_{n1-\tau 1,n2-\tau 2}$$

In matrix form, correlation may be presented as multiplication of matrix of a template and matrix of an analysed image. The task is to reduce complexity and time during matching of template and image in binary image analysis.

3. DIRECT METHOD OF CORRELATION COMPUTATION BASED ON MATRIX FACTORISATION

Peculiarity of the binary image correlation processing in matrix form is the lack of multiplication operations for the direct method of correlation computation such as multiplying to 1 does not change the pixel and multiplying to 0 gives 0 in result. Consequently, the main problem of the image processing computational complexity reducing is the decreasing quantity of the summing up operations. From this point of view, effect of the utilisation of various techniques based on Fourier transformation for binary images processing come down, because some data has complex form and needs the operation of the multiplication calculation for image processing. To calculate correlation coefficients with the help of Fourier transform, it is necessary to do the following:

- calculate direct Fourier transform for vectors (image strings), i.e. multiply image strings to matrixes of direct transformations;
- calculate point multiplication of obtained vectors;
- calculate reverse Fourier trnsform – multiply obtained vectors to matrix of reverse transformation.

Direct method of the matrix multiplication calculation does not need the operation of multiplication calculation, but leads to considerable computational complexity, because of a huge number of the summing up operations to be calculated. For reducing the number of the summing up operations we apply the following:

- in matrix form, the calculation of correlation function for two images may be presented as consequent computation of correlation function for object template matrix strings and initial image matrix strings (in vector-matrix multiplication form);

- for reduction of vector-matrix multiplication, computational complexity of this procedure can be presented in terms of matrix factorisation. According to this method, matrix may be represented as a multiplication of n submatrixes, each of them has only two non-zero elements in each string.

Usually, an object template is available for processing. Preliminary factorisation of binary template matrix is performed before processing and does not conduce to computational complexity increasing. Factorisation of binary matrix is performed by the following rule.

Any matrix A with size N*m consisting 0 and 1 can represented in the form of multiplication:

$$A=D*B$$

where B – matrix with size N1*m, N1<N obtained from matrix A after deleting repeated and inverse strings, D – matrix with size N*N1 with elements:

$$d_{ij} = \begin{cases} 1, \text{ if } b_j = a_i \\ 0, \text{ if } b_j = -a_i \\ \text{in other cases info simbols are absent} \end{cases}$$

So, binary image of template

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

can be presented as the following:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \\ & & 1 \\ & 1 & \\ 1 & & \end{bmatrix} \cdot \begin{bmatrix} 1 & & & 1 & \\ & 1 & & & 1 \\ & & 1 & 0 & \end{bmatrix} \cdot \begin{bmatrix} 1 & & 1 & & \\ & 1 & 1 & & \\ 0 & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & & & & \\ 0 & 1 & & & & \\ & & 1 & 1 & & \\ & & & & 1 & 1 \\ & & & & 1 & 0 \end{bmatrix}$$

Traditional method of multiplying of vector to this matrix requires 30 operations of addition/subtraction type. Consequent multiplying of vector to these matrix requires only 11 such operations

So, the first reduction of computational complexity can be achieved by factorisation of template matrix.

Consider now correlational search of the given template in the matrix:

$$\begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&0&0&0&0&0&1&1&1&1&1&1 \\ 0&0&0&1&1&0&0&0&0&0&0&1&1&1&1&0 \\ 0&0&1&1&1&1&0&0&0&0&0&0&1&1&0&0 \\ 0&1&1&1&1&1&1&0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0 \end{bmatrix}$$

It is performed for vertical block of image N*m (m – quantity of columns) by multiplying strokes of image block to matrix of template. Analysis of obtained correlation coefficients is performed and decision is taken for object presence in the image. Then, shift into one column is performed and procedure is repeated. For search of m*m template in N*N image, we need (N-m) shifts. Maximal necessary number of operations for correlational search can be defined as

$$K = k \cdot N \cdot (N - m + 1)$$

where k – number of operations for multiplying of image stroke to template (for traditional method we need k= $m \cdot (m-1)$;

For a given matrix, traditional multiplying of vector to matrix gives 5280 operations of addition/subtraction type

$$K=30*16*11=5280$$

By using matrix factorisation, for the given example, we need only 1936 operations:

$$K=11*16*11=1936$$

Further computational complexity reducing for vector-matrix multiplication is possible if the matrix block dividing procedure for factorisation will be optimised. For doing it, the vector-matrix multiplication calculation is presented as two intermediate procedures: internal (input vector and block-matrix multiplication) and external (intermediate data summing up). The result of the preliminary investigations is that the minimum number of operations will be got through dividing the square matrix dimension 2^n*2^n by blocks of dimension (n-1) [5]. In general, for matrix dimension N*N ($2^n<N<2^{n+1}$) optimal block size is:

$$m=\lceil \log_2 N+0.5 \rceil-1$$

where: $\lceil * \rceil$ - is the minimum nearest number integer.

General quantity of the plus/mines operations for vector-matrix calculation computation, considering ($\lfloor n/2 \rfloor$<m<n), must be computed [7]:

$$C \approx K \cdot \frac{N}{m} + \left(N \cdot \left\lfloor \frac{N}{m} \right\rfloor - 1 \right)$$

where: $n=\log_2 N$

4. COMPARISON AND DISCUSSION

Number of operations for calculation of correlation function for various sizes of binary matrixes and several methods is presented in Tabl.1. These results were obtained in assumption that the complexity of the one operation multiplication is equivalent to three plus/minus operations complexity.

Table 1. Comparison of number of operations for correlation function calculation in some algorithms

N*N (matrix size)	Rader-Brenner algorithm	Nussbaumer algorithm	Nussbaumer polynomial algorithm and split-radix FFT	Our algorithm, (based on matrix factorisation)
16*16	8 192	6 628	6 844	1 792
32*32	46 336	35 780	35 516	10 240
64*64	241 152	194 820	174 780	68 812
128*128	1 190 912	948 100	830 140	463 667
256*256	5 675 008	4 739 204	3 844 796	3 099 852
512*512	26 357 760	21 521 028	14 777 284	21 495 808

The comparison of the results shows that the computational complexity of our approach to the correlation computation is more attractive in comparison with other approaches for templates with size less than 256*256. For templates of bigger size, it is necessary to perform correlation processing dividing them into fragments of smaller size.

In general, image processing for direct correlation function computation utilises the principles and realises all advantages of maximum likelihood method. Another advantage is the absence of the trigonometric and complex multiplications and summing up what is required for various Fourier transformations. This absence allows the use of ordinary microprocessors without floating-point unit and permits to use the standard procedure of creating recursive structure for programming only with main program cycle.

5.CONCLUSION

A novel algorithm for time reduction in binary image processing, namely for computation of correlation between image and object template has been proposed. This algorithm is based on direct computation of vector-matrix multiplication with utilisation of binary matrix factorisation approach. Comparison with other algorithms is given and it is shown that our approach allows to reduce time and complexity of this task.

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