A Special Type of Multi-Dimensional Integral Transform with Fox *H*-Function in Lebesgue-type Weighted Spaces

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Abstract

This paper is devoted to the study of multi-dimensional integral transform with Fox H-function in kernels in weighted spaces of Lebesgue measurable functions in the domain with positive coordinates. By using the technique of the multidimensional Mellin transformation, mapping properties such as the boundedness, the range, the representations of the considered transform are established. Research results generalize those obtained earlier for the corresponding one-dimensional transformation.

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Ethics declarations

The authors of this work declare that they have no conflicts of interest.

Additional information

Dedicated to memory of the prominent mathematician Valeriy Vyacheslavovich Katrakhov on the occasion of his 75th jubilee

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